JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

GRADE 12

2026 LAST PSUH

SOLUTION TO QUESTION PAPER BANK

PAPER 1

&

PAPER 2

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INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

SECTION A: PAPER 1

ALGEBRA, EQUATIONS AND INEQUALITIES

Paper A

1.1.1

$3x^2 + 5x = 0$

$x(3x + 5) = 0$

$x = 0 \quad \text{or} \quad x = -\frac{5}{3}$

1.1.2

$4x^2 + 3x - 5 = 0$

$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(4)(-5)}}{2(4)}$

$x = 0{,}80 \quad \text{or} \quad x = -1{,}55$

1.1.3

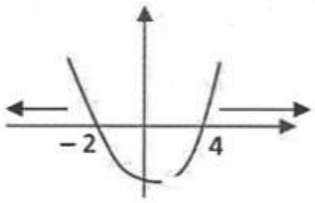
$(x-1)^2 - 9 \geq 0$

$x^2 - 2x - 8 \geq 0$

$(x-4)(x+2) \geq 0$

$x = 4 \quad \text{or} \quad x = -2$

$x \leq -2 \quad \text{or} \quad x \geq 4$



1.1.4

$5^{2x} - 5^x = 0$

$5^x(5^x - 1) = 0$

$5^x \neq 0 \quad \text{or} \quad 5^x = 1$

$x = 0$

OR/OF

$5^{2x} = 5^x$

$2x = x$

$x = 0$

1.1.5

$$\frac{x}{\sqrt{20-x}}=1$$
$$x=\sqrt{20-x}$$
$$x^2=20-x$$
$$x^2+x-20=0$$
$$(x+5)(x-4)=0$$
$$x=4 \text{ or } x\neq -5$$

1.2

$$2x^2-y^2=7 \quad \ldots. (1)$$
$$x+y=9 \quad \ldots. (2)$$
$$y=9-x$$
$$2x^2-(9-x)^2=7$$
$$2x^2-81+18x-x^2=7$$
$$x^2+18x-88=0$$
$$(x+22)(x-4)=0$$
$$x=-22 \text{ or } x=4$$
$$y=31 \text{ or } y=5$$

OR/*OF*

$$2x^2-y^2=7 \quad \ldots. (1)$$
$$x+y=9 \quad \ldots. (2)$$
$$x=9-y$$
$$2(9-y)^2-y^2=7$$
$$2(81-18y+y^2)-y^2-7=0$$
$$162-36y+2y^2-y^2-7=0$$
$$y^2-36y+155=0$$
$$(y-31)(y-5)=0$$
$$y=31 \text{ or } y=5$$
$$x=-22 \text{ or } x=4$$

1.3

$$P\times T=(1-a)(1+a)(1+a^2)(1+a^4)...(1+a^{512})$$
$$P\times T=(1-a^2)(1+a^2)(1+a^4)...(1+a^{512})$$
$$P\times T=(1-a^4)(1+a^4)...(1+a^{512})$$
$$P\times T=(1-a^8)...(1+a^{512})$$
$$P\times T=(1-a^{512})(1+a^{512})$$
$$=1-a^{1024}$$

Paper B

1.1.1

$x^2 + x - 12 = 0$

$(x-3)(x+4) = 0$

$x = 3 \text{ or } x = -4$

1.1.2

$3x^2 - 2x = 6$

$3x^2 - 2x - 6 = 0$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$

$x = 1{,}79 \text{ or } x = -1{,}12$

1.1.3

$\sqrt{2x+1} = x - 1$

$2x + 1 = (x-1)^2$

$2x + 1 = x^2 - 2x + 1$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \text{ or } x = 4$

$x \neq 0 \text{ or } x = 4$

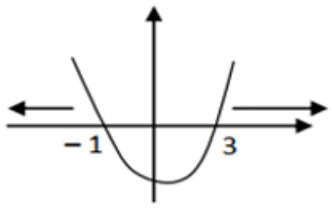
1.1.4

$x^2 - 2x > 3$

$x^2 - 2x - 3 > 0$

$(x-3)(x+1) > 0$

CV's: $x = -1 \; ; x = 3$



$x < -1 \text{ or } x > 3$

1.2

$$\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \qquad (1)$$

$$x+2=2y \qquad \ldots \qquad (2)$$

$$x=2y-2$$

$$\frac{1}{2y-2}+\frac{1}{y}=1$$

$$y+2y-2=2y^2-2y$$

$$2y^2-5y+2=0$$

$$(2y-1)(y-2)=0$$

$$y=\frac{1}{2} \quad \text{or} \quad y=2$$

$$x=-1 \quad \text{or} \quad x=2$$

OR/*OF*

$$\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \qquad (1)$$

$$x+2=2y \qquad \ldots \qquad (2)$$

$$y=\frac{x}{2}+1$$

$$\frac{1}{x}+\frac{1}{\frac{x}{2}+1}=1$$

$$\frac{1}{x}+\frac{2}{x+2}=1$$

$$x+2+2x=x^2+2x$$

$$x^2-x-2=0$$

$$(x+1)(x-2)=0$$

$$x=-1 \quad \text{or} \quad x=2$$

$$y=\frac{1}{2} \quad \text{or} \quad y=2$$

1.3

$2^{m+1} + 2^m = 3^{n+2} - 3^n$
$2^m(2+1) = 3^n(3^2 - 1)$
$2^m(3) = 3^n(8)$
$2^m(3) = 3^n(2^3)$
$\therefore m = 3 \text{ and } n = 1$
$\therefore m + n = 4$

OR/*OF*

$2^{m+1} + 2^m = 3^{n+2} - 3^n$
$2^m(2+1) = 3^n(3^2 - 1)$
$2^m(3) = 3^n(8)$
$2^m(3) = 3^n(2^3)$
$2^{m-3} = 3^{n-1}$
Only true if $m - 3 = 0$ and $n - 1 = 0$
$\therefore m + n = 4$

Paper C

1.1.1

$(3x-6)(x+2)=0$

$x=2 \quad \text{or} \quad x=-2$

1.1.2

$2x^2-6x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{6\pm\sqrt{(-6)^2-4(2)(1)}}{2(2)}$$

$x=2{,}82 \quad \text{or} \quad x=0{,}18$

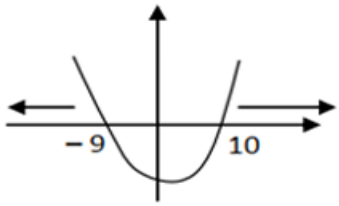
1.1.3

$x^2-90>x$

$x^2-x-90>0$

$(x+9)(x-10)>0$

CV: $x=-9 \quad \text{or} \quad x=10$



$x<-9 \quad \text{or} \quad x>10$

OR/OF

$(-\infty;-9) \text{ or } (10;\infty)$

1.1.4

$x-7\sqrt{x}=-12$

$x+12=7\sqrt{x}$

$(x+12)^2=\left(7\sqrt{x}\right)^2$

$x^2+24x+144=49x$

$x^2-25x+144=0$

$(x-16)(x-9)=0$

$x=16 \text{ or } x=9$

OR/*OF*

$x-7\sqrt{x}+12=0$

$\left(\sqrt{x}-3\right)\left(\sqrt{x}-4\right)=0 \quad \text{or} \quad \text{let } \sqrt{x}=k$

$\sqrt{x}=3 \text{ or } \sqrt{x}=4$

$x=9 \quad \text{or} \quad x=16$

1.2

$2x - y = 2$

$y = 2x - 2 \quad \ldots\ldots\ldots(1)$

$xy = 4 \quad \ldots\ldots\ldots(2)$

(1) in (2):

$x(2x-2) = 4$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad \text{or} \quad x = -1$

$y = 2 \qquad y = -4$

OR/*OF*

$2x - y = 2$

$x = \frac{1}{2}y + 1 \quad \ldots\ldots\ldots(1)$

$xy = 4 \quad \ldots\ldots\ldots(2)$

(1) in (2):

$y\left(\frac{1}{2}y + 1\right) = 4$

$\frac{1}{2}y^2 + y - 4 = 0$

$y^2 + 2y - 8 = 0$

$(y+4)(y-2) = 0$

$y = -4 \quad \text{or} \quad y = 2$

$x = -1 \qquad x = 2$

OR/OF

$2x-y=2 \quad \ldots\ldots\ldots(1)$

$y=\frac{4}{x} \qquad \ldots\ldots\ldots(2)$

(2) in (1):

$2x-\frac{4}{x}=2$

$2x^2-2x-4=0$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2 \quad \text{or} \quad x=-1$

$y=2 \qquad y=-4$

OR/OF

$2x-y=2 \quad \ldots\ldots\ldots(1)$

$x=\frac{4}{y} \qquad \ldots\ldots\ldots(2)$

(2)in (1):

$2\left(\frac{4}{y}\right)-y=2$

$8-y^2-2y=0$

$y^2+2y-8=0$

$(y+4)(y-2)=0$

$y=-4 \quad \text{or} \quad y=2$

$x=-1 \qquad x=2$

1.3

$2.5^n-5^{n+1}+5^{n+2}=2.5^n-5^n.5^1+5^n.5^2$

$=5^n(2-5+25)$

$=5^n(22)$

$2\left(5^n(11)\right)$

OR/OF

Any integer multiplied by an even number will be even

1.4

$$\frac{3^{y+1}}{32} = \sqrt{96^x}$$

$$\frac{3^{y+1}}{2^5} = (96)^{\frac{x}{2}}$$

$$3^{y+1}.2^{-5} = 2^{\frac{5x}{2}}.3^{\frac{x}{2}}$$

$$-5 = \frac{5x}{2}$$

$$\therefore x = -2$$

$$y+1 = \frac{x}{2}$$

$$y+1 = \frac{-2}{2}$$

$$\therefore y = -2$$

Paper D

1.1.1

$x = -\frac{1}{2}$ OR $x = 0$

1.1.2

$5x^2 + 3x - 1 = 0$

$\therefore x = \frac{-3 \pm \sqrt{3^2 - 4(5)(-1)}}{2(5)}$

$x = 0,24$ or $x = -0,84$

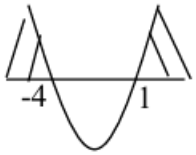
1.1.3

$2x^2 + 6x - 8 \geq 0$

$2(x+4)(x-1) \geq 0$ OR $(2x+8)(x-1) \geq 0$

$CV: x = -4$ or $x = 1$

$\therefore x \leq -4$ or $x \geq 1$



1.1.4

$\sqrt{3x+13} = x+1$

$3x + 13 = x^2 + 2x + 1$

$0 = x^2 - x - 12$

$0 = (x-4)(x+3)$

$\therefore x = 4$ OR $x \neq -3$

1.1.5

$3^{(2x+2)} + 8.3^x - 1 = 0$

$9.3^{2x} + 8.3^x - 1 = 0$

$(9.3^x - 1)(3^x + 1) = 0$

$3^x = \frac{1}{9}$ OR $3^x \neq -1$

$\therefore x = -2$

OR

$3^{(2x+2)} + 8.3^x - 1 = 0$

$9.3^{2x} + 8.3^x - 1 = 0$

Let $k = 3^x$

$9k^2 + 8k - 1 = 0$

$(9k-1)(k+1) = 0$

$k = \frac{1}{9}$ or $k = -1$

$\therefore 3^x = \frac{1}{9}$ *OR* $3^x \neq -1$

$\therefore x = -2$

1.2

$x - 2y - 3 = 0$ *and* $x^2 - 3xy + y^2 = 11$

$x = 2y + 3$ .....(1)

$x^2 - 3xy + y^2 - 11 = 0$ .....(2)

$(2y+3)^2 - 3(2y+3)y + y^2 - 11 = 0$

$4y^2 + 12y + 9 - 6y^2 - 9y + y^2 - 11 = 0$

$y^2 - 3y + 2 = 0$

$(y-2)(y-1) = 0$

$y = 2$ or $y = 1$

$x = 7$ or $x = 5$

1.3

$f(x) = x^2 - 5x + c$

$b^2 - 4ac = 41$

$(-5)^2 - 4(1)(c) = 41$

$-4c = 16$

$c = -4$

OR

$\left(\frac{5+\sqrt{41}}{2}\right)^2 - 5\left(\frac{5+\sqrt{41}}{2}\right) + c = 0$

$c = -4$

OR

$\left(\frac{5-\sqrt{41}}{2}\right)^2 - 5\left(\frac{5-\sqrt{41}}{2}\right) + c = 0$

$c = -4$

Paper E

1.1	1.1.1	$x^2-121=0$ $(x-11)(x+11)=0$ $x=11$ or / *of* $x=-11$
	1.1.2	$x^2+3=5x$ $x^2-5x+3=0$ $x=\frac{5\pm\sqrt{(-5)^2-4(1)(3)}}{2(1)}$ $x=\frac{5\pm\sqrt{13}}{2}$ $x=4,30$ or/*of* $x=0,70$
	1.1.3	$(x-1)(x-5)\le 12$ $x^2-6x-7\le 0$ $(x+1)(x-7)\le 0$ CV: $x=-1$ or / *of* $x=7$  Sol/*Opl*: $-1\le x\le 7$

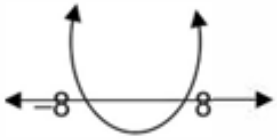
	1.1.4	$2^x + 2^{x+2} = 40$ $2^x + 2^x.2^2 = 40$ $2^x(1+4) = 40$ $2^x = 8$ $2^x = 2^3$ $x = 3$
	1.1.5	$\sqrt{10-x^2} - x + 2 = 0$ $\sqrt{10-x^2} = x - 2$ $10 - x^2 = x^2 - 4x + 4$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or / *of* $x \neq -1$
1.2		$y - x = 2$ $x^2 + 2xy - 4 = 0$ $y = x + 2$ $x^2 + 2x(x+2) - 4 = 0$ $3x^2 + 4x - 4 = 0$ $(3x-2)(x+2) = 0$ $x = \frac{2}{3}$ or/*of* $x = -2$ $y = \frac{8}{3}$ or/*of* $y = 0$
		OR/*OF* $y - x = 2$ $x^2 + 2xy - 4 = 0$ $x = y - 2$ $(y-2)^2 + 2y(y-2) - 4 = 0$ $y^2 - 4y + 4 + 2y^2 - 4y - 4 = 0$ $3y^2 - 8y = 0$ $y(3y-8) = 0$ $y = 0$ or/*of* $y = \frac{8}{3}$ $x = -2$ or /*of* $x = \frac{2}{3}$

1.3

$$f(x) = x(x+a) - b$$
$$f(a) = 0$$
$$a(a+a) - b = 0$$
$$2a^2 - b = 0$$
$$b = 2a^2$$

$$g(x) = 2x^2 + b - dx$$
$$g(b) = 0$$
$$2b^2 + b - bd = 0$$
$$b(1-d) = 2b^2$$
$$1 - d = 2b$$
$$1 - d = 2(2a^2)$$
$$1 - d = 4a^2$$

Paper F

1.1.1	$x(x+6)=0$ $x=0$ or $x=-6$
1.1.2	$3x^2+8x=-2$ $3x^2+8x+2=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-8\pm\sqrt{(8)^2-4(3)(2)}}{2(3)}$ $x=0,23$ or $x=-2,90$
1.1.3	$x^2-64\le 0$ $(x+8)(x-8)\le 0$ Critical Values: -8 and 8  $-8\le x\le 8$ OR $[-8;8]$
1.1.4	$x\sqrt{x+5}+1=x$ $\sqrt{x+5}=x-1$ $\left(\sqrt{x+5}\right)^2=(x-1)^2$ $x+5=x^2-2x+1$ $x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4$ or $x=-1$ $\therefore x=4$ but $x\neq -1$

1.2 $6x+5xy-5y=8$ and $x+y=2$

$x=2-y...(3)$

$6(2-y)+5(2-y)y-5y=8$

$12-6y+10y-5y^2-5y=8$

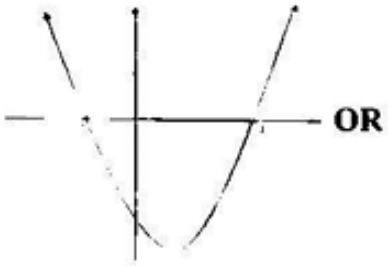
$5y^2+y-4=0$

$(5y-4)(y+1)=0$

$y=\frac{4}{5}$ or $y=-1$

$x=\frac{6}{5}$ or $x=3$

Paper G

1.1.1	$x=-3 \text{ or } x=\frac{2}{5}$
1.1.2	$7x^2+5x-8=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(5)\pm\sqrt{(5)^2-4(7)(-8)}}{2(7)}$ $x=-1,48 \text{ or } x=0,77$
1.1.3	$\frac{7}{x^2-2x-8}>0$ $\therefore x^2-2x-8>0$ $(x-4)(x+2)>0$  **OR**  $x<-2 \text{ or } x>4$
1.1.4	$3^{x+2}=42-5.3^x$ $3^x.3^2+5.3^x=42$ $3^x(9+5)=42$ $3^x=3^1$ $x=1$
1.1.5	$-\sqrt{5x-1}=5-x$ $\left(-\sqrt{5x-1}\right)^2=(5-x)^2$ $5x-1=25-10x+x^2$ $x^2-15x+26=0$ $(x-2)(x-13)=0$ $x\neq 2 \text{ or } x=13$

1.2

$$x = 3 + y$$

$$(3+y)^2 - y(3+y) = 2y^2 + 7$$

$$9 + 6y + y^2 - 3y - y^2 - 2y^2 - 7 = 0$$

$$2y^2 - 3y - 2 = 0$$

$$(2y+1)(y-2) = 0$$

$$y = -\frac{1}{2} \text{ or } y = 2$$

$$x = 3 + \left(-\frac{1}{2}\right) \text{ or } x = 3 + 2$$

$$x = \frac{5}{2} \text{ or } x = 5$$

OR

$$y = x - 3$$

$$x^2 - x(x-3) = 2(x-3)^2 + 7$$

$$x^2 - x^2 + 3x - 2x^2 + 12x - 18 - 7 = 0$$

$$2x^2 - 15x + 25 = 0$$

$$(2x-5)(x-5) = 0$$

$$x = \frac{5}{2} \text{ or } x = 5$$

$$y = \frac{5}{2} - 3 \text{ or } y = 5 - 3$$

$$y = -\frac{1}{2} \text{ or } y = 2$$

1.3

$$2x^2 + px - p^2 = 0$$

$$\Delta = b^2 - 4ac$$

$$= (p)^2 - 4(2)(-p^2)$$

$$= p^2 + 8p^2$$

$$= 9p^2$$

$$= (3p)^2$$

$\therefore$ The roots are rational

OR

$$2x^2 + px - p^2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(p) \pm \sqrt{(p)^2 - 4(2)(-p^2)}}{2(2)}$$

$$= \frac{-p \pm \sqrt{9p^2}}{4}$$

$$= \frac{-p \pm 3p}{4}$$

$$= \frac{p}{2} \text{ or } -p$$

$\therefore$ The roots are rational

Paper H

1.1.1

$x^2+2x+1=0$

$(x+1)(x+1)=0$

$x+1=0$ or/*of* $x+1=0$

$x=-1$ or/*of* $x=-1$

OR/*OF*

$x^2+2x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(2)\pm\sqrt{(2)^2-4(1)(1)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{0}}{2}$$

$x=-1$ or / *of* $x=-1$

1.1.2

$x(5x-3)=1$

$5x^2-3x-1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(5)(-1)}}{2(5)}$$

$$x=\frac{3\pm\sqrt{29}}{10}$$

$\therefore x=-0,24$ or / *of* $x=0,84$

1.1.3

$2x+3>x^2$

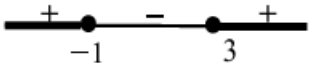
$-x^2+2x+3>0$

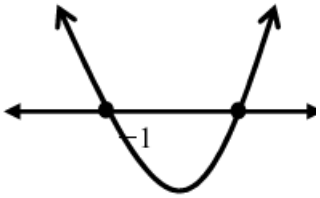
$x^2-2x-3<0$

critical values/*kritieke waardes*

$(x+1)(x-3)=0$

$x=-1$ or/*of* $x=3$





$-1<x<3,\ x\in\mathbf{R}$

OR/*OF*

$x\in(-1\ ;\ 3),\ x\in\mathbf{R}$

1.1.4

$\sqrt{7x-12}-x=0$

$\sqrt{7x-12}=x$

$\left(\sqrt{7x-12}\right)^2=(x)^2$

$7x-12=x^2$

$x^2-7x+12=0$

$(x-3)(x-4)=0$

$\therefore\ x=3$ or / *of* $x=4$

1.1.5

$\left(\frac{1}{6}\right)^{3x+2}.216^{3x}=\frac{1}{216}$

$6^{-3x-2}\times 6^{9x}=6^{-3}$

$\therefore\ 6x-2=-3$

$6x=-1$

$x=-\frac{1}{6}$

1.2	$x+y=3\ldots\ldots\ldots\ldots\ldots\ldots(1)$ $x^2+2y^2=18\ldots\ldots(2)$ From/$Vanaf(1)$: $y=-x+3\ldots\ldots\ldots(3)$ (3)into/$in(2)$: $x^2+2(-x+3)^2=18$ $x^2+2(x^2-6x+9)-18=0$ $x^2+2x^2-12x+18-18=0$ $3x^2-12x=0$ $3x(x-4)=0$ $3x=0$ or/$ofx-4=0$ $x=0$ or/$ofx=4$ $y=3$ or/$ofy=-1$
1.3	$\left(\sqrt{5}\right)^x-\left(\sqrt[3]{2}\right)^y=17$ $(5)^{\frac{1}{2}x}-(2)^{\frac{1}{3}y}=25-8$ $(5)^{\frac{1}{2}x}-(2)^{\frac{1}{3}y}=5^2-2^3$ $\therefore \frac{1}{2}x=2$ and/en $\frac{1}{3}y=3$ $\Rightarrow x=4$ and/en $y=9$ $P=(4\times9)=36$

Paper I

1.1.1	$(3x-6)(x+5)=0$ $x=2 \quad or \quad x=-5$
1.1.2	$7x^2+4x-9=0$ $x=\frac{-(4)\pm\sqrt{(4)^2-4(7)(-9)}}{2(7)}$ $x=0.88 \quad or \quad x=-1.46$
1.1.3	$(3-x)(x+7)<0$ $cv: x=3 \quad or \quad x=-7$ $x<-7 \quad or \quad x>3$ or $-(x-3)(x+7)<0$ $(x-3)(x+7)>0$ $x=3 \quad or \quad x=-7$ $x<-7 \quad or \quad x>3$
1.1.4	$3\sqrt{x+3}-5=x$ $3\sqrt{x+3}=x+5$ $\left(3\sqrt{x+3}\right)^2=(x+5)^2$ $9(x+3)=x^2+10x+25$ $9x+27=x^2+10x+25$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2 \quad or \quad x=1$ *Both valid answers*

1.2

$y = 3x + 2 \ldots\ldots (1)$

$\frac{1}{y} + \frac{3}{x} = -4$

$\frac{1}{3x+2} + \frac{3}{x} = -4$

$x + 3(3x+2) = -4(x)(3x+2)$

$x + 9x + 6 = -12x^2 - 8x$

$12x^2 + 18x + 6 = 0$

$2x^2 + 3x + 1 = 0$

$(2x+1)(x+1) = 0$

$x = -\frac{1}{2} \ or \ \ x = -1$

$y = 3\left(\frac{-1}{2}\right) + 2 \qquad or \ \ y = 3(-1) + 2$

$y = \frac{1}{2} \qquad\qquad or \ y = -1$

1.3

$\frac{2^x}{8} = \frac{3^y}{3}$

$\frac{2^x}{2^3} = \frac{3^y}{3}$

$2^{x-3} = 3^{y-1}$

$x - 3 = 0 \qquad y - 1 = 0$

$x = 3 \qquad\quad y = 1$

$x + y = 3 + 1 = 4$

Paper J

1.1.1

$$(x+2)(x-2)=5$$
$$\therefore x^2-4=5$$
$$\therefore x^2=9$$
$$\therefore x=\pm 3$$

1.1.2

$$\frac{1}{2}x^2-2x-1=0$$

Method 1

$$\frac{1}{2}x^2-2x-1=0$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4\left(\frac{1}{2}\right)(-1)}}{2\left(\frac{1}{2}\right)}=2\pm\sqrt{6}$$

$$\therefore x=4,45 \quad \text{or} \quad x=-0,45$$

Method 2

$$\frac{1}{2}x^2-2x-1=0$$
$$\therefore x^2-4x-2=0$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-2)}}{2(1)}=\frac{4\pm\sqrt{24}}{2}$$

$$\therefore x=4,45 \quad \text{or} \quad x=-0,45$$

1.1.3

$5 < x(7-2x)$

$2x^2 - 7x + 5 < 0$

$(2x-5)(x-1) < 0$

C/V: 1 & $\frac{5}{2}$

$1 < x < \frac{5}{2}$

But since $x > 2$

$\therefore 2 < x < \frac{5}{2}$

1.1.4

$3^x - 1 = \sqrt{2.3^{x+1} + 10}$

$(3^x - 1)^2 = 2.3^{x+1} + 10$

$3^{2x} - 2.3^x + 1 - 2.3^x.3^1 - 10 = 0$

$3^{2x} - 8.3^x - 9 = 0$

$(3^x - 9)(3^x + 1) = 0$

$3^x = 9$ or $3^x = -1$

$\therefore 3^x = 3^2$

$\therefore x = 2$

1.2.1

For real and equal roots $\Delta = 0$

$\therefore p^2 - 2p + 1 = 0$

$(p-1)(p-1) = 0$

$\therefore p = 1$

1.2.2

$x = 2 \pm \sqrt{1^2 - 2(1) + 1}$

$\therefore x = 2$

$\therefore (x-2)(x-2) = 0$

$\therefore x^2 - 4x + 4 = 0$

$\therefore b = -4$

$\therefore c = 4$

1.3

$$\frac{2^{2y}}{2^x} = 2^{-1}$$

$$2^{2y-x} = 2^{-1}$$

$$\therefore x = 2y + 1$$

$$x^2 + y^2 = 29$$

$$(2y+1)^2 + y^2 - 29 = 0$$

$$5y^2 + 4y - 28 = 0$$

$$(y-2)(5y+14) = 0$$

$$\therefore y = 2 \text{ or } y = -\frac{14}{5}$$

$$x = 5 \text{ or } x = -\frac{23}{5}$$

PATTERNS, SEQUENCES AND SERIES

Paper A

2.1.1

$r = \frac{1}{2}$

Yes, because $-1 < \frac{1}{2} < 1$

2.1.2

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{4}{1-\frac{1}{2}}$$

$$\therefore S_\infty = 8$$

2.2

$$\sum_{p=k}^{10} 3^{p-1} = 3^{k-1} + 3^{k+1-1} + 3^{k+2-1} + \ldots + 3^9$$

$$= 3^{k-1} + 3^k + 3^{k+1} + \ldots 3^9$$

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}$$

$$29\,520 = \frac{3^{k-1}\left(3^{11-k} - 1\right)}{3-1}$$

$$3^{10} - 3^{k-1} = 59\,040$$

$$3^{k-1} = 9$$

$$k - 1 = 2$$

$$\therefore k = 3$$

3.1.1

3 ; 7 ; 12 ; 18

V V V

First diff: 4 ; 5 ; 6

V V

Second diff: 1 ; 1

$$2a = 1$$

$$a = \frac{1}{2}$$

$$3a + b = 4$$

$$3\left(\frac{1}{2}\right) + b = 4$$

$$b = \frac{5}{2}$$

$$a + b + c = 3$$

$$\frac{1}{2} + \frac{5}{2} + c = 3$$

$$c = 0$$

	$T_n = \frac{1}{2}n^2 + \frac{5}{2}n$
3.1.2	$13\,527 = \frac{1}{2}n^2 + \frac{5}{2}n$ $n^2 + 5n - 27\,054 = 0$ $(n-162)(n+167) = 0$ $n = 162$ or $n = -167$ $T_{161} = 13\,363$ $13\,527 - 13363 = 164$ 164 must be added.
3.2.1	$T_n = 8 + (n-1)(3)$ $T_n = 3n + 5$ $41 = 3n + 5$ $36 = 3n$ $n = 12$
3.2.2a	$P_{41} = 12$
3.2.2b	$P_8 = a + 7d = 1$ $P_{11} = a + 10d = 2$ $3d = 1$ $d = \frac{1}{3}$ $a + 7\left(\frac{1}{3}\right) = 1$ $a = -\frac{4}{3}$ **OR/*OF*** $n = 3P_n + 5$ $1 = 3P_1 + 5$ $-4 = 3P_1$ $P_1 = -\frac{4}{3}$ **OR/*OF*** $T_n = 3n + 5$ $1 = 3n + 5$ $n = -\frac{4}{3}$ $\therefore P_1 = -\frac{4}{3}$

Paper B

2.1.1	$(10+t)+(t-2)+(t+4)$ $\frac{t-2}{t+10}=\frac{t+4}{t-2}$ $(t-2)^2=(t+4)(t+10)$ $t^2-4t+4=t^2+14t+40$ $-18t=36$ $\therefore t=-2$
2.1.2	$8;-4;\ldots$ $r=\frac{-4}{8}=-\frac{1}{2}$ $T_{25}=8\left(-\frac{1}{2}\right)^{24}$ $T_{25}=\left(\frac{1}{2}\right)^{21}$ or $T_{25}=(2)^{-21}$ or $T_{25}=(8)^{-7}$ or $T_{25}=(128)^{-3}$ or $T_{25}=(2\ 097\ 152)^{-1}$
2.1.3	$S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{8}{1-\left(-\frac{1}{2}\right)}$ $S_\infty=\frac{16}{3}=5,33$
2.2.1	$T_{14}-T_6$ $=4\times 8$ $=32$ **OR/*OF*** $T_{14}-T_6$ $=(4(14)-1)-(4(6)-1)$ $=55-23$ $=32$ **OR/*OF*** $T_{14}-T_6$ $=(4(k+13)-1)-(4(k+5)-1)$ $=4k+51-(4k+19)$ $=32$

2.2.2

$n = 118 - k$

$T_{117} = 467$

$S_{118-k} = \frac{118-k}{2}\left[4k - 1 + 467\right]$

$26\ 675 = (118-k)\left[2k + 233\right]$

$26\ 675 = 236k + 27494 - 2k^2 - 233k$

$2k^2 - 3k - 819 = 0$

$(k - 21)(2k + 39) = 0$

$\therefore k = 21 \text{ or } k \neq -\frac{39}{2}$

OR/*OF*

$n = 118 - k$

$S_{118-k} = \frac{118-k}{2}\left[2(4k-1) + (118-k-1)(4)\right]$

$26\ 675 = \frac{118-k}{2}\left[8k - 2 + 468 - 4k\right]$

$53\ 350 = (118-k)\left[4k + 466\right]$

$53\ 350 = 472k + 54\ 988 - 4k^2 - 466k$

$4k^2 - 6k - 1638 = 0$

$2k^2 - 3k - 819 = 0$

$(k - 21)(2k + 39) = 0$

$\therefore k = 21 \text{ or } k \neq -\frac{39}{2}$

OR/OF

$T_1 = 3 \qquad T_2 = 7$

$S_{117} - S_{k-1} = 26\ 675$

$S_{117} = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{117}{2}\left[2(3) + (116)4\right]$

$= 27\ 495$

$\therefore S_{k-1} = 27\ 495 - 26\ 675 = 820$

$820 = \frac{n}{2}\left[2(3) + 4n - 4\right]$

$0 = 2n^2 + n - 820$

$(2n + 41)(n - 20) = 0$

$\therefore n = 20$

$\therefore k - 1 = 20$

$k = 21$

3.1	36 ; 71 ; 104 ; 135 ; 164 35 33 31 29 -2 $T_5 = 164$
3.2	$2a = -2$ $a = -1$ $3(-1) + b = 35$ $b = 38$ $-1 + 38 + c = 36$ $c = -1$ $T_n = -n^2 + 38n - 1$
3.3	$n = \frac{-38}{2(-1)} = 19$ $T_{19} = -(19)^2 + 38(19) - 1 = 360$ **OR/*OF*** $T'_n = -2n + 38 = 0$ $\therefore n = 19$ $T_{19} = -(19)^2 + 38(19) - 1 = 360$
3.4	$\frac{n+3}{2} = 19$ $n + 3 = 38$ $n = 35$ **OR/*OF*** $-n^2 + 38n - 1 = 104$ $n^2 - 38n + 105 = 0$ $(n-35)(n-3) = 0$ $n = 35$

Paper C

2.1.1

$a = 14$

$T_6 = 14r^5 = 448$

$r^5 = 32$

$\therefore r = 2$

2.1.2

$T_n = 14(2)^{n-1}$

$S_n = \dfrac{14(2^6 - 1)}{2 - 1}$

$S_6 = 882$

$114\,674 - 882 = 113\,792$

$113\,792 = 896\left(2^n - 1\right)$

$128 = 2^n$

$n = 7$

OR/OF

$S_n = \dfrac{a(r^n - 1)}{r - 1}$

$114\,674 = \dfrac{14(2^n - 1)}{2 - 1}$

$8\,191 = 2^n - 1$

$2^n = 8\,192$

$n = \log_2 8\,192$

$n = 13$

$\therefore$ 7 more terms must be added to the first 6 terms.

2.1.3

$r = \dfrac{1}{2}$ OR $448r^5 = 14$

$\therefore r = \dfrac{1}{2}$

$S_\infty = \dfrac{a}{1 - r}$

$S_\infty = \dfrac{448}{1 - \frac{1}{2}}$

$S_\infty = 896$

2.2

$$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$$

$$T_1=\frac{1}{6} \qquad T_2=\frac{1}{3}+\frac{1}{6}=\frac{3}{6}$$

$$d=\frac{3}{6}-\frac{1}{6}=\frac{1}{3}$$

$$\frac{121}{6}=\frac{n}{2}\left[2\left(\frac{1}{6}\right)+(n-1)\left(\frac{1}{3}\right)\right]$$

$$\frac{121}{3}=n\left[\frac{1}{3}+\frac{1}{3}n-\frac{1}{3}\right]$$

$$\frac{121}{3}=\frac{1}{3}n^2$$

$$121=n^2$$

$$n=11$$

$$\therefore k=10$$

OR/*OF*

$$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$$

$$a=\frac{1}{6}$$

$$l=\frac{1}{3}k+\frac{1}{6}$$

$$n=k+1$$

$$S_n=\frac{n}{2}[a+l]$$

$$\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{6}+\frac{1}{3}k+\frac{1}{6}\right]$$

$$\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{3}k+\frac{1}{3}\right]$$

$$\frac{121}{6}=\frac{k+1}{2}\left[\frac{k+1}{3}\right]$$

$$\frac{121}{6}=\frac{(k+1)^2}{6}$$

$$k+1=\pm\sqrt{121}$$

$$k+1=11$$

$$k=10$$

3.1	$3a+b=7$ $3+b=7$ $b=4$ **OR/OF** $T_2-T_1=7$ $4+2b+9-(1+b+9)=7$ $b=4$
3.2	$T_n=n^2+4n+9$ $T_{60}=(60)^2+4(60)+9$ $=3849$
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; … Common 2nd difference: 2 $T_p=2p+5$ **OR/*OF*** First difference: 7 ; 9 ; 11 ; … $T_n=a+(n-1)d$ $T_p=7+(p-1)(2)$ $T_p=2p+5$
3.4	$157=2p+5$ $p=76$ $\therefore$ Between T_{76} and T_{77}

OR

$$T_{n+1}-T_n=157$$
$$(n+1)^2+4(n+1)+9-(n^2+4n+9)=157$$
$$n^2+2n+1+4n+4+9-n^2-4n-9=157$$
$$2n=152$$
$$n=76$$

$\therefore$ Between T_{76} and T_{77}

Paper D

2.1.1	$T_n = 2n+3$
2.1.2	$93 = 2n+3$ $90 = 2n$ $45 = n$
2.1.3	$50+70+90+...+930$ $S_{45} = \frac{45}{2}[2(50)+(45-1)(20)]$ $S_{45} = \text{R}22\,050$ Total raised = R22 050 **OR/*OF*** $50+70+90+...+930$ $S_{45} = \frac{45}{2}[50+930]$ $S_{45} = \text{R}22\,050$ Total raised = R22 050 **OR/*OF*** $5+7+9+...+93$ $S_{45} = \frac{45}{2}[2(5)+(45-1)(2)]$ $S_{45} = 2\ 205\text{km}$ $S_{45} = \text{R}22\,050$ Total raised = R22 050 **OR/*OF*** $5+7+9+...+93$ $S_{45} = \frac{45}{2}[5+93]$ $S_{45} = 2\ 205\text{km}$ $S_{45} = \text{R}22\,050$ Total raised = R22 050
2.2.1 a)	$T_1 = (2)^{1+2}$ $T_1 = 8$
2.2.1 b)	$r = 2$
2.2.2	$T_{20} = (2)^{20+2}$ $T_{20} = 2^{22} = (2^2)^{11}$ $= 4^{11}$

2.2.3	$\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$ $S_{\infty}=\frac{a}{1-r}$ $=\frac{\frac{1}{8}}{1-\frac{1}{2}}$ $\therefore S_{\infty}=\frac{1}{4}$
2.2.4	$8+4^2+32+4^3+\ldots+4^{11}+\ldots$ $S_{21}-S_{10}=\frac{8\left(2^{21}-1\right)}{2-1}-\frac{16\left(4^{10}-1\right)}{4-1}$ $=16\ 777\ 208-5\ 592\ 400$ $=11\ 184\ 808$ **OR/*OF*** $8+32+128+$ $S_{11}=\frac{8\left(4^{11}-1\right)}{4-1}$ $\therefore S_{11}=11\ 184\ 808$

3.1	$14 \;;\; 9 \;;\; 6 \;;\; 5 \;;\; \ldots.$ $-5 \quad -3 \quad -1$ $2 \quad 2$ $2a = 2 \qquad 3(1) + b = -5 \qquad 1 - 8 + c = 14$ $a = 1 \qquad b = -8 \qquad c = 21$ $\therefore T_n = n^2 - 8n + 21$
3.2	$T_n = -5 + (n-1)(2)$ $T_n = 2n - 7$ $2n - 7 = 33$ $\therefore n = 20$ $\therefore T_{21} = (21)^2 - 8(21) + 21$ $T_{21} = 294$ **OR/*OF*** $\therefore T_{n+1} - T_n = (n+1)^2 - 8(n+1) + 21 - n^2 + 8n - 21$ $n^2 + 2n + 1 - 8n - 8 + 21 - n^2 + 8n - 21 = 33$ $2n - 7 = 33$ $\therefore n = 20$ $\therefore T_{21} = (21)^2 - 8(21) + 21$ $T_{21} = 294$
3.3	$T_7 = T_1 = 14$ $\therefore 14 + m \geq 0$ $m \geq -14$ And $T_6 = T_2$ $\therefore 9 + m < 0$ $m < -9$ $\therefore -14 \leq m < -9$

Paper E

2.1.1	209 ; 186
2.1.2	321 ; 290 ; 261 ; 234 1st diff: −31 −29 −27 2nd diff: 2 2 $2a = 2 \quad 3a + b = -31 \quad a + b + c = 321$ $a = 1 \quad 3(1) + b = -31 \quad 1 + (-34) + c = 321$ $b = -34 \quad c = 354$ $T_n = n^2 - 34n + 354$
2.1.3	$n^2 - 34n + 354 = 74$ $n^2 - 34n + 280 = 0$ $(n-14)(n-20) = 0$ $n = 14$ or $n = 20$
2.1.4	$f'(n) = 0$ $2n - 34 = 0$ $2n = 34$ $n = 17$ Term 17 will have the smallest value **OR/OF** $n = \frac{-b}{2a}$ $n = \frac{34}{2}$ $n = 17$ Term 17 will have the smallest value **OR/OF** $n = \frac{14 + 20}{2} = 17$ Term 17 will have the smallest value

2.2.1

$$a=\frac{5}{8} \quad ; \quad r=\frac{1}{2} \quad ; \quad n=21$$

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$S_{21}=\frac{\frac{5}{8}\left(1-\left(\frac{1}{2}\right)^{21}\right)}{1-\frac{1}{2}}$$

$$=1{,}2499...$$

$$=1{,}25$$

2.2.2

$$T_n>\frac{5}{8192}$$

$$ar^{n-1}>\frac{5}{8192}$$

$$\frac{5}{8}\left(\frac{1}{2}\right)^{n-1}>\frac{5}{8192}$$

$$\left(\frac{1}{2}\right)^{n-1}>\frac{1}{1024}$$

$$\left(\frac{1}{2}\right)^{n-1}>\left(\frac{1}{2}\right)^{10} \quad or \quad 2^{-n+1}>2^{-10}$$

$$\therefore n-1<10 \qquad -n+1>-10$$

$$n<11 \qquad n<11$$

$$\therefore n=10 \qquad \therefore n=10$$

OR/OF

8 ; 16 ; 32 ; … ; 8192

$$8.2^{n-1}<8192$$

$$2^{n-1}<1024$$

$$2^{n-1}<2^{10}$$

$$n-1<10$$

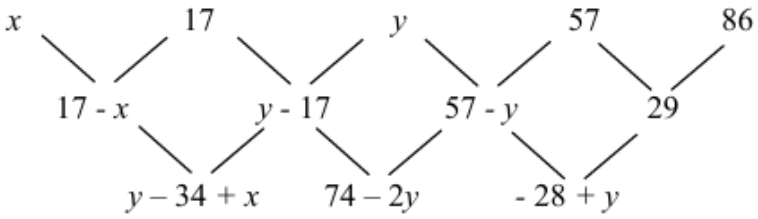
$$n<11$$

$$\therefore n=10$$

3.1	$\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$ $=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}\right)-\left(\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}+\frac{1}{9}\right)$ $=1-\frac{1}{9}$ $=\frac{8}{9}$
3.2	$\left(\frac{1}{3}\times\frac{2}{3}\right)+\left(\frac{2}{3}\times\frac{2}{3}\right)+\left(1\times\frac{2}{3}\right)+....+\left(4\times\frac{2}{3}\right)$ $=\frac{2}{9}+\frac{4}{9}+\frac{2}{3}+....+\frac{8}{3}$ $a=\frac{2}{9}$ and $d=\frac{2}{3}-\frac{4}{9}=\frac{2}{9}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ OR $S_n=\frac{n}{2}(a+l)$ $S_{12}=\frac{12}{2}\left[2\left(\frac{2}{9}\right)+(12-1)\frac{2}{9}\right]$ $S_{12}=\frac{12}{2}\left(\frac{2}{9}+\frac{8}{3}\right)$ $=\frac{52}{3}\text{m}^2$ $=\frac{52}{3}\text{m}^2$ $\therefore$ for both sides $=2\times\frac{52}{3}=\frac{104}{3}=34{,}67\text{m}^2$ **OR/OF** $\frac{2}{9}\times(1+2+3+4+5+6+7+8+9+10+11+12)\times 2$ $=34{,}67\text{ m}^2$ **OR/OF** $T_1=\frac{2}{9}\times 12=\frac{8}{3}$ $l=\frac{2}{9}\times 1=\frac{2}{9}$ $2S_{12}=2\left(\frac{12}{2}\right)\left(\frac{8}{3}+\frac{2}{9}\right)$ $=34{,}67\text{ m}^2$

Paper F

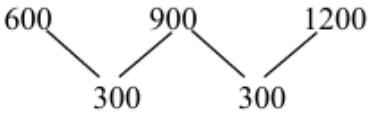
2.1



$$74-2y=-28+y$$
$$3y=102$$
$$y=34$$

$$y-34+x=74-2y$$
$$34-34+x=74-2(34)$$
$$x=6$$

2.2.1



$$T_{15}=600+(14)(300)$$
$$=4800m \qquad OR \qquad =4,8km$$

OR

$$T_n=300n+300$$
$$T_{15}=300(15)+300$$
$$=4800\text{ m}$$

2.2.2

$$S_{15}=\frac{15}{2}[2(600)+(14)(300)]$$
$$=40\ 500m$$
$$=40,5km$$

2.2.3	$T_{180} = 600 + (179)(300)$ $= 54\ 300m$ $= 54,3km$ She will be eligible to run the Comrades. **OR** $42000 = 600 + (n-1)(300)$ $41700 = 300n$ $n = 139\ days$ She would be able to run 42 km in less than 6 months. **OR ANY OTHER CORRECT INTERPRETATION, WITH THE CORRECT CONCLUSION**
2.3	$96\ ;\ 91\ ;\ 86 \quad (d = -5)$ $S_{21} = \frac{21}{2}\ [2(96) + 20(-5)]$ $= 966$
2.4.1	$p = \frac{a}{1-r}$ $r = 1 - \frac{a}{p}$ $-1 < 1 - \frac{a}{p} < 1$ $-2 < -\frac{a}{p} < 0$ $-2p < -a < 0$ $\therefore 0 < a < 2p \quad OR \quad a \in (0;2p)$
2.4.2	$a = \frac{p}{4}$ $p = \frac{a}{1-r}$ $\therefore a = p(1-r)$ $\therefore p(1-r) = \frac{p}{4}$ $1-r = \frac{1}{4}$ $r = \frac{3}{4}$

Paper G

2.1.1	Yes / *Ja* ; $-1<\frac{1}{2}<1$
2.1.2	$S_\infty = \frac{a}{1-r}$ $= \frac{36}{1-\frac{1}{2}}$ $= 72\ \text{cm}^2$
2.1.3	Side length series / *Sylengte reeks*: $6 + 3\sqrt{2} + 3\ ; \ldots \Rightarrow r = \frac{1}{2}$ $T_n = 6\left(\frac{1}{\sqrt{2}}\right)^{n-1} = \frac{3}{8}$ $\left(\frac{1}{\sqrt{2}}\right)^{n-1} = \left(\frac{1}{16}\right)$ $\quad \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{256}\right)$ $\left(\frac{1}{\sqrt{2}}\right)^{n-1} = \left(\frac{1}{\sqrt{2}}\right)^8$ *OR / OF* $\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^8$ $\therefore\ n-1 = 8$ $n = 9$ **OR/*OF*** Area series / *Oppervlakte reeks* side length / *sylengte* = $\frac{3}{8}$ $\Rightarrow$ Area / *Oppervlakte* = $\frac{9}{64}$ $T_n = 36\left(\frac{1}{2}\right)^{n-1} = \frac{9}{64}$ $\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{256}\right)$ $\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^8$ $\therefore\ n-1 = 8$ $n = 9$

2.1.4 Series of ONE diagonal / *Reeks van EEN hoeklyn*:

$6\sqrt{2} + 6 + 3\sqrt{2} + 3 + \ldots$

$\Rightarrow r = \frac{1}{\sqrt{2}}$

$S_n = \frac{a\left(1-r^n\right)}{1-r}$

$= \frac{6\sqrt{2}\left(1-\left(\frac{1}{\sqrt{2}}\right)^{10}\right)}{1-\frac{1}{\sqrt{2}}}$

$= 28,07 \text{ units / } eenhede$

$\because$ there are two diagonals: $\because$ ***daar twee hoeklyne is***:

$\Rightarrow 2 \times 28,07 = 56,14 \text{units / } eenhede$

OR/*OF*

Series of TWO diagonals / *Reeks van TWEE hoeklyne*:

$12\sqrt{2} + 12 + 6\sqrt{2} + 6 + \ldots$

$\Rightarrow r = \frac{1}{\sqrt{2}}$

$S_n = \frac{a\left(1-r^n\right)}{1-r}$

$= \frac{12\sqrt{2}\left(1-\left(\frac{1}{\sqrt{2}}\right)^{10}\right)}{1-\frac{1}{\sqrt{2}}}$

$= 56,13 \text{ units / } eenhede$

2.2.1	$a+11d=14$ .....(1) $a+6d=4$ .....(2) $5d=10$ $d=2$ $a+6(2)=4$ $a=-8$
2.2.2	$T_n=-8+(n-1)(2)$ $=2n-10$ $2n-10=8$ $2n=18$ $n=9$ $T_9=8$, is the additive inverse of first term *is die optellingsinverse van die eerste term*
2.3	$T_1=3\ ;\ T_2=7\ ;\ T_3=11 \Rightarrow d=4$ $T_{100}=399$ $S_n=\frac{n}{2}(a+l)$ $19995=\frac{101-k}{2}[(4k-1)+399]$ $39990=(101-k)(4k+398)$ $4k^2-6k-208=0$ $2k^2-3k-104=0$ $k=8 \quad or \quad k\neq-\frac{13}{2}$

3.1.1	**3 ; 12 ; 33 ; 66 ; T_5** **9 ; 21 ; 33 ; 45** **12 ; 12 ; 12** $T_5 = 111$
3.1.2	$T_9 = 6(9)^2 - 9(9) + 6$ $= 411$ $T_9 - T_5 = 411 - 111$ $= 300$
3.2	$T_n = an^2 + bn + 1$ $T_4 = 27 \Rightarrow 16a + 4b + 1 = 27$ $T_3 - T_2 = 8 \Rightarrow 5a + b = 8$ $20a + 4b = 32$ $16a + 4b = 26$ $4a = 6$ $a = \frac{3}{2}$ $5\left(\frac{6}{4}\right) + b = 8$ $b = 8 - \frac{15}{2}$ $b = \frac{1}{2}$

Paper H

2.1.1 $T_5 = -14$

2.1.2

$$T_n = a + (n-1)d$$

$$6 + (n-1)(-5) = -239$$

$$6 - 5n + 5 = -239$$

$$5n = 250$$

$$n = 50$$

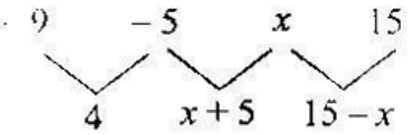
$$S_n = \frac{n}{2}[a + l] \qquad \text{OR} \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{50} = \frac{50}{2}[6 + (-239)] \qquad S_{50} = \frac{50}{2}[2(6) + (50-1)(-5)]$$

$$= -5825 \qquad S_{50} = 25[-233]$$

$$= -5825$$

2.2.1



$$x + 5 - 4 = 15 - x - (x + 5)$$

$$3x = 9$$

$$x = 3$$

2.2.2	1st differences: 4; 8; 12 2nd difference $=4=2a$ $\therefore a=2$ $3a+b=4$ $3(2)+b=4$ $b=-2$ $a+b+c=-9$ $2-2+c=-9$ $c=-9$ $T_n=2n^2-2n-9$
2.2.3	$T_n=2n^2-2n-9=2(n^2-n-4)-1$ $2(n^2-n-4)$ is **even** An even number minus 1 is **an odd number** **OR** $T_n=2n^2-2n-9=2(n^2-n)-9$ $2(n^2-n)$ is even; 9 is odd. An even number minus an odd number is an odd number

3.1.1

$$4+4.p^{-1}+4.p^{-2}+\ldots=6$$

$$4+\frac{4}{p}+\frac{4}{p^2}+\ldots=6$$

$$\therefore r=\frac{1}{p}$$

$$S_\infty=\frac{a}{1-r}$$

$$\therefore \frac{4}{1-\frac{1}{p}}=6$$

$$4=6\left(1-\frac{1}{p}\right)$$

$$4=6-\frac{6}{p}$$

$$\frac{6}{p}=2$$

$$p=3$$

3.1.2 $4;\ 4.3^{-1};\ 4.3^{-2}$ **OR** $4;\ \frac{4}{3};\ \frac{4}{3^2}$ **OR** $4;\ \frac{4}{3};\ \frac{4}{9}$

3.2.1 $5^4=625$

3.2.2

$$5^1+5^2+5^3+\ldots\ldots+5^{11}$$

$$S_n=\frac{a\left(r^n-1\right)}{r-1}$$

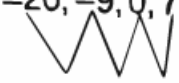
$$S_{11}=\frac{5\left(5^{11}-1\right)}{5-1}$$

$$=61\,035\,155$$

Paper I

2.1.1

$-20;-9;0;7;\ldots$



$11 \quad 9 \quad 7$



$-2 \quad -2$

$2a = -2 \qquad 3(-1) + b = 11$

$-1 + 14 + c = -20$

$a = -1 \qquad b = 14$

$c = -7$

$\therefore T_n = -n^2 + 14n - 7$

2.1.2

$n = \dfrac{-b}{2a}$

$= \dfrac{-14}{2(-1)}$

$n = 7$

$\therefore T_7 = -(7)^2 + 14(7) - 7$

$= 42$

3.1.1	$13;8;3;...$ $a=13 \quad \text{and} \quad d=-5$ $T_n=a+(n-1)d$ $T_{50}=13+(50-1)-5$ $T_{50}=57$
3.1.2	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{50}=\frac{50}{2}[2(13)+(50-1)(-5)]$ $S_{50}=-5475$
3.2	$S_n=a+(a+d)+(a+2d)+...+(l-2d)+(l-d)+l...(1)$ $S_n=l+(l-d)+(l-2d)+...(a+2d)+(a+d)+a...(2)$ $2S_n=(a+l)+(a+1)+(a+l)+...(a+l)+(a+l)+(a+l)$ $\therefore 2S_n=n(a+l)$ $\therefore S_n=\frac{n}{2}(a+l)$ $\therefore S_n=\frac{n}{2}[a+a+(n-1)d]$ $\therefore S_n=\frac{n}{2}[2a+(n-1)d]$
3.3.1	$3+m+\frac{m^2}{3}+\frac{m^3}{9}+...$ $r=\frac{m}{3}$ $-1<r<1$ $-1<\frac{m}{3}<1$ $-3<m<3$

3.3.2

$$S_\infty = \frac{a}{1-r}$$

$$\frac{27}{7} = \frac{3}{1-\frac{m}{3}}$$

$$27 - \frac{27m}{3} = 21$$

$$27 - 9m = 21$$

$$6 = 9m$$

$$\therefore m = \frac{6}{9} = \frac{2}{3} = 0,67$$

3.4

$$\sum_{r=1}^{n} 5.2^{1-r} = 5 + \frac{5}{2} + \frac{5}{4} + \ldots$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\frac{630}{64} = \frac{5\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}$$

$$\frac{63}{64} = 1-\left(\frac{1}{2}\right)^n$$

$$\therefore \left(\frac{1}{2}\right)^n = \frac{1}{64}$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$$

$$n = 6$$

Paper J

2.1.1

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(7) + (20-1)(5)]$$

$$S_{20} = 1\,090$$

2.1.2

$$T_n = 7 + (n-1)(5)$$

$$T_n = 5n + 2$$

$$S_{75} - S_{20} = 14\,400 - 1\,090 = 13\,310$$

$$\sum_{n=1}^{75}(5n+2) - \sum_{n=1}^{20}(5n+2) = 13\,310$$

$$\therefore \sum_{n=21}^{75}(5n+2) = 13\,310$$

OR/*OF*

$$T_{21} + T_{22} + T_{23} + \ldots + T_{75} = 14\,400 - 1\,090$$

$$107 + 112 + 117 + \ldots. = 13\,310$$

$$T_n = 102 + 5n$$

$$\therefore \sum_{1}^{55}(5n+102) = 13\,310$$

2.2.1

$$T_n = 2n - 1$$

98^{th} first difference = $2(98) - 1$

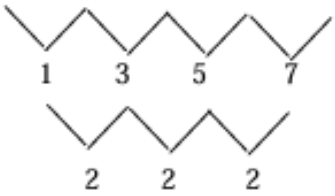
$= 195$

Quadratic: $T_{98} = 9\,632 - 195$

$= 9\,437$

2.2.2

28 ; 29 ; 32



$3a + b = 1$

$2a = 2$ $\quad 3(1) + b = 1$

$a = 1$ $\quad b = -2$

$9a + 3b + c = 32$

$T_3 = 1(3)^2 - 2(3) + c = 32$

$c = 29$

OR/*OF*

$1 - 2 + c = 28$

$c = 29$

$\therefore T_n = n^2 - 2n + 29$

3.1	2^{nd} circle: $\frac{6}{2} = 3\text{ cm}$ 3^{rd} circle: $\frac{3}{2} = 1{,}5\text{ cm}$ **OR/*OF*** $3^{rd}\text{ circle } = 6\left(\frac{1}{2}\right)^2 = 1{,}5$
3.2	$36\pi\ ;\ 9\pi\ ;\ \frac{9}{4}\pi\ ;\ ...$ $r = \frac{1}{4}\ \ ;\ \ a = 36\pi$ $S_{10} = \frac{36\pi\left(\left(\frac{1}{4}\right)^{10} - 1\right)}{\frac{1}{4} - 1}$ $S_{10} = 150{,}80$
3.3	$6\ ;\ 3\ ;\ \frac{3}{2}\ldots$ $\text{radius} = \frac{1}{2}\left(\frac{3}{128}\right) = \frac{3}{256}$ $\frac{3}{256} = 6\left(\frac{1}{2}\right)^{n-1}$ $\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$ $\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$ **or/*of*** $n-1 = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{512}\right)$ $\therefore n-1 = 9$ $\quad\quad$ $\therefore n-1 = 9$ $\therefore n = 10$ $\quad\quad$ $\therefore n = 10$

OR

$12\ ;\ 6\ ;\ 3\ ;\ \ldots$

$\text{diameter} = \dfrac{3}{128}$

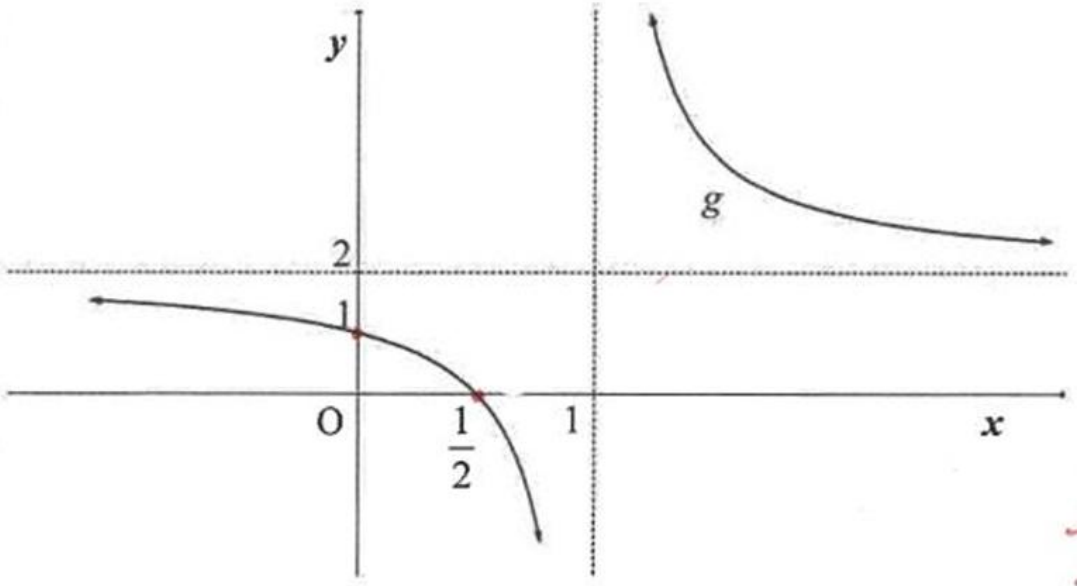
$\dfrac{3}{128} = 12\left(\dfrac{1}{2}\right)^{n-1}$

$\dfrac{1}{512} = \left(\dfrac{1}{2}\right)^{n-1}$

$\left(\dfrac{1}{2}\right)^{9} = \left(\dfrac{1}{2}\right)^{n-1}$ **or/*of*** $n-1 = \log_{\left(\frac{1}{2}\right)}\left(\dfrac{1}{512}\right)$

$\therefore n-1=9$ $\qquad$ $\therefore n-1=9$

$\therefore n=10$ $\qquad$ $\therefore n=10$

4.1	$x = 1$ $y = 2$
4.2	
4.3	$x < \frac{1}{2}$ or $x > 1$ **or/of** $\left(-\infty;\frac{1}{2}\right)$ or $(1;\infty)$
4.4	$y = -(x-1)+2$ $y = -x+3$

OR

$y-2 = -(x-1)$
$y = -x+3$

OR

$y = -x+c$
$2 = -(1)+c$
$c = 3$
$\therefore y = -x+3$

5.1	$P'(2;4)$
5.2	$f(x)=\log_a x$ $2=\log_a 4$ $a^2=4$ $a=2$
5.3	$y=2^x$
5.4	$1=\log_2 x$ $\therefore x=2 \qquad T(2;1)$ RT = 2 units P'T = 3 units Area of $\Delta RTP' = \frac{1}{2}.RT.TP'$ $= \frac{1}{2}\times 2\times 3 = 3 \text{ units}^2$

6.1	$y \geq -4 \quad \text{or} \quad y \in [-4;\infty)$
6.2	$x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3 \quad \text{or} \quad x = -1$ $\therefore \text{E}(3\,;\,0) \quad \text{and} \quad \text{D}(-1\,;\,0)$
6.3	$\text{P}(0\,;-3)$ $\therefore m_g = 1$ $\therefore g(x) = x - 3$
6.4	$f(x) > g(x)$ $x < 0 \quad \text{or} \quad x > 3$
6.5	$\text{Distance} = -x^2 + 2x + 3 - x + 3 = -x^2 + x + 6$ $\text{D}' = -2x + 1 = 0 \qquad \text{or/of} \qquad x = -\frac{b}{2a} = -\frac{1}{2(-1)}$ $\therefore x = \frac{1}{2} \qquad\qquad \therefore x = \frac{1}{2}$ $\text{D}\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$ $= \frac{25}{4} = 6{,}25$

6.6

$f'(x) = m_g$

$2x - 2 = 1$

$x = \frac{3}{2}$

Point on f and k: $\left(\frac{3}{2}; \frac{-15}{4}\right)$

$k(x) = g(x) - n \quad \therefore -\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$

$\therefore n = 2\frac{1}{4} = \frac{9}{4} = 2{,}25$

OR

$f(x) = k(x)$

$x^2 - 2x - 3 = x - 3 - n$

$x^2 - 3x + n = 0$

$\Delta = b^2 - 4ac$

$= (-3)^2 - 4(1)(n)$

To touch: $\Delta = 0$

$0 = 9 - 4n$

$4n = 9$

$n = \frac{9}{4} = 2{,}25$

Paper B

4.1	$y=-4$
4.2	$x-$ intercept: $0=2^x-4$ $4=2^x$ $x=2$ $\therefore \mathrm{B}(2;0)$
4.3	$y=2^0-4=-3$ $\therefore \mathrm{A}(0;-3)$ $y=mx+c$ $m=\frac{3}{2}$ $k(x)=\frac{3}{2}x-3$
4.4	$k(1)=\frac{3}{2}(1)-3=\frac{-3}{2}$ $f(1)=2^1-4=-2$ Vertical distance $=-\frac{3}{2}-(-2)=\frac{1}{2}$ units
4.5	$g(x)=f(x)+4$ $g(x)=2^x\ ;\ x\in[-2;4)$
4.6	Range of g: $y\in\left[\frac{1}{4};16\right)$ Domain of g^{-1}: $x\in\left[\frac{1}{4};16\right)$ or/*of* $\frac{1}{4}\le x<16$
4.7	$g: y=2^x$ $g^{-1}: x=2^y$ $g^{-1}(x)=\log_2 x,\ x\in\left[\frac{1}{4};16\right)$

5.1	$(1\ ;\ 8)$
5.2	$y=-\frac{1}{2}(0-1)^2+8$ $=7\frac{1}{2}$ $C\left(0;\frac{15}{2}\right)$
5.3	$8=\frac{d}{1}$ $\therefore d=8$
5.4	$y\in R\ ;\ y\neq 0$
5.5	$-3\leq x<0$ or $x\geq 5$ **OR/*OF*** $x\in[-3\ ;\ 0)\cup[5;\infty)$
5.6	$-2x+k=\frac{8}{x}$ $-2x^2+kx-8=0$ $\Delta=(k)^2-4(-2)(-8)$ $k^2-64<0$ $CV: k=8\ ;\ k=-8$ $\therefore -8<k<8$ **or/of** $k\in(-8;8)$
	OR/*OF* $g'(x)=h'(x)$ $-\frac{8}{x^2}=-2$ $-8=-2x^2$ $x=\pm 2$ $y=\pm 4$ $\therefore$ B(2 ; 4) and A(– 2 ; – 4) For tangents: $h(x)=-2x+k$ or $h(x)=-2x+k$ $4=-2(2)+k$ $\quad -4=-2(-2)+k$ $k=8$ $\quad k=-8$ $\therefore -8<k<8$ or/*of* $k\in(-8;8)$

5.7

$$h(x) = -2x + 8$$

$$-2x + 8 = \frac{8}{x}$$

$$-2x^2 + 8x = 8$$

$$-2x^2 + 8x - 8 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

$$\therefore x = 2$$

$$f(2) = \frac{15}{2}$$

$$h(2) = 4$$

$$4 = \frac{15}{2} + t$$

$$\therefore t = -\frac{7}{2}$$

OR

$f(2)=\frac{15}{2}$

Tangent point of contact (2 ; 4)

$\therefore 4=-\frac{1}{2}(2-1)^2+8+t$

$4=\frac{15}{2}+t$

$\therefore t=-\frac{7}{2}$

OR/*OF*

$g(x)=8x^{-1}$

$g'(x)=-8x^{-2}$

$-2=-8x^{-2}$

$\frac{1}{4}=\frac{1}{x^2}$

$x=2$

$y=\frac{8}{2}=4$

$R(2;4)$

$y=-\frac{1}{2}(x-1)^2+8+t$

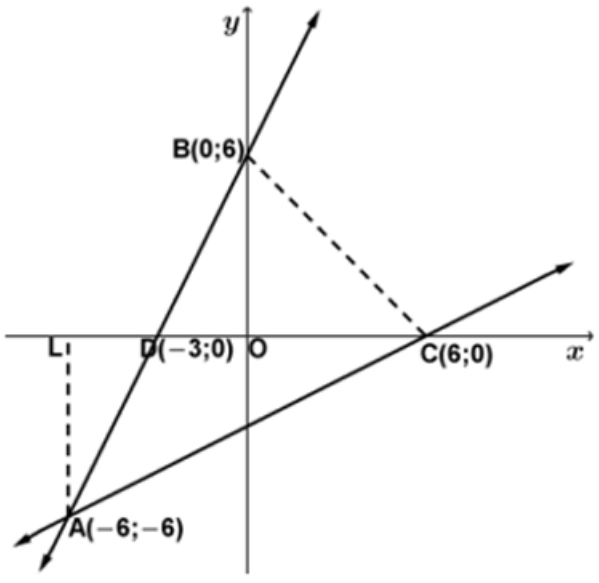
$4=-\frac{1}{2}(2-1)^2+8+t$

$t=-\frac{7}{2}$

Paper C

4.1.1	$p=-1$ and $q=2$
4.1.2	$\frac{1}{x-1}+2=0$ $-2x+2=1$ $x=\frac{1}{2}$ $\left(\frac{1}{2};0\right)$
4.1.3	$x=\frac{1}{2}-3$ $=\frac{-5}{2}$
4.1.4	$y=x+t$ $2=1+t$ $t=1$
4.1.5	$-2\leq\frac{1}{x-1}$ $\frac{1}{x-1}+2\geq 0$ $\therefore x\leq\frac{1}{2}$ or $x>1$ **OR/*OF*** $x\in\left(-\infty;\frac{1}{2}\right]$ or $(1;\infty)$
4.2.1	$y=-5$

4.2.2	$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$ $f(2) = 2^2 - 4(2) - 5 = -9$ $\therefore \mathrm{D}(2;-9)$ **OR/*OF*** $f'(x) = 2x - 4$ $2x - 4 = 0$ $x = 2$ $f(2) = 2^2 - 4(2) - 5 = -9$ $\therefore \mathrm{D}(2;-9)$
4.2.3	$q = -5$ $-9 = a(2)^2 - 5$ $-4 = 4a$ $a = -1$ $\therefore g(x) = -2^x - 5$
4.2.4	$y \in (-\infty;-5)$ OR $y < -5; y \in R$
4.2.5	$k < -9$

5.1	$g(x) = 2x + 6$ $y = 6$
5.2	$y = 2x + 6$ $x = 2y + 6$ $y = \frac{1}{2}x - 3$
5.3	$\frac{1}{2}x - 3 = 2x + 6$ $x - 6 = 4x + 12$ $3x = -18$ $x = -6$ $A(-6;-6)$ **OR/*OF*** $2x + 6 = x$ $x = -6$ $y = -6$
5.4	$AB = \sqrt{(6)^2 + (12)^2}$ $= \sqrt{180} = 6\sqrt{5} = 13{,}42$
	

5.5

$$BC = \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$$

$$AB = AC = \sqrt{180} \qquad \text{symmetry of } g \text{ and } g^{-1}$$

$$\perp h = (\sqrt{180})^2 - \left(\frac{\sqrt{72}}{2}\right)^2$$

$$= \sqrt{162} = 9\sqrt{2}$$

$$\text{area of } \Delta ABC = \frac{1}{2} BC \times h$$

$$= \frac{1}{2} \times \sqrt{72} \times \sqrt{162} = 54 \text{ units}^2$$

OR/*OF*

$$\tan B\hat{D}C = 2$$

$$\therefore B\hat{D}C = 63{,}43°$$

$$\tan D\hat{C}A = \frac{1}{2}$$

$$\therefore D\hat{C}A = 26{,}57°$$

$$\therefore D\hat{A}C = 36{,}86° \qquad \text{(ext angle triangle)}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left(\sqrt{180}\right)\left(\sqrt{180}\right)\sin 36{,}86°$$

$$= 53{,}99 \text{ units}^2$$

OR

$$\text{Area of } \Delta ABC = \text{Area of } \Delta BDC + \text{Area of } \Delta ADC$$

$$= \frac{1}{2} DC.BO + \frac{1}{2} DC.height$$

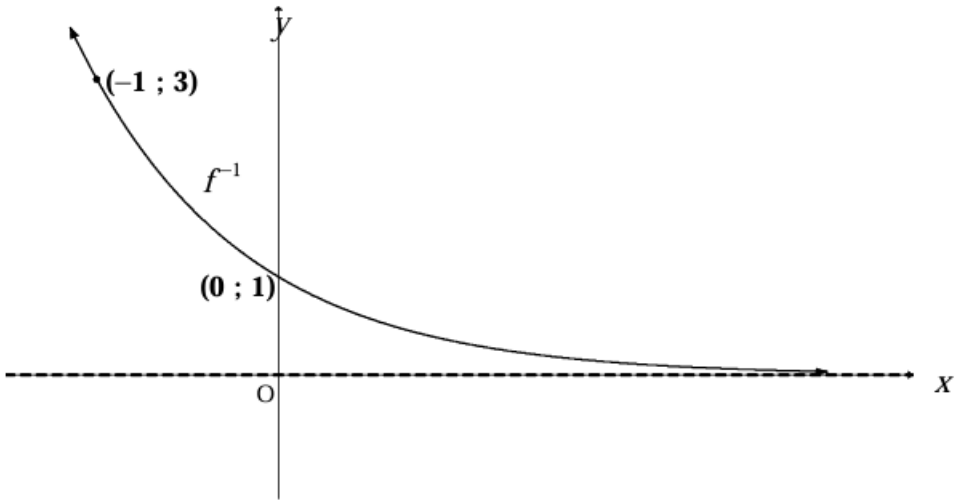
$$= \frac{1}{2}(9)(6) + \frac{1}{2}(9)(6)$$

$$= 54 \text{ units}^2$$

Paper D

4.1	$t = \log_{\frac{1}{3}} 3$ $t = -1$
4.2	$A(1;0)$
4.3	$f(x) = \log_{\frac{1}{3}} x$ $y = \log_{\frac{1}{3}} x$ $x = \log_{\frac{1}{3}} y$ $y = \left(\frac{1}{3}\right)^x = 3^{-x}$
4.4	$y = 0$

4.5



4.6	(4 ; 3) is a point on h $0 < y < 3$ or $y \in (0 ; 3)$

5.1	$x \in R,\ x \neq 3$
5.2	$y \leq 8$
5.3.1	$3 < x \leq 5$ **OR/*OF*** $x \in (3;5]$
5.3.2	$x < 1$ or $x > 5$ **OR/*OF*** $x \in (-\infty;1)$ or $x \in (5;\infty)$
5.4	$y = a(x-3)^2 + 8$ $6 = a(5-3)^2 + 8$ $-2 = 4a$ $\therefore a = -\frac{1}{2}$ $y = -\frac{1}{2}(x-3)^2 + 8$ $y = -\frac{1}{2}(x^2 - 6x + 9) + 8$ $y = -\frac{1}{2}x^2 + 3x + \frac{7}{2}$

5.5

$$y=-\frac{1}{2}x^2+3x+\frac{7}{2}$$

$$x^2-6x-7=0$$

$$(x-7)(x+1)=0$$

$$x=7 \quad \text{or} \quad x=-1$$

$$\text{M}(-1\,;0)$$

$$0=\frac{-4}{x-3}+8$$

$$-8x+24=-4$$

$$x=\frac{7}{2}$$

$$\text{T}\left(\frac{7}{2}\,;0\right)$$

$$\text{MT}=\frac{7}{2}+1=\frac{9}{2}=4,5$$

5.6

$$f(x)=-\frac{1}{2}x^2+3x+\frac{7}{2}$$

$$f'(x)=-x+3$$

$$m=f'(5)$$

$$=-5+3$$

$$=-2$$

$$6=-2(5)+c$$

$$c=16$$

$$\therefore y=-2x+16$$

6.1 $(-p\,;0)$

6.2 $g(x)=x+c$

$q=1+c$

$$f(x)=\frac{a}{x+c}+1+c$$

Point $(0\,;c)$

$$c=\frac{a}{0+c}+1+c$$

$$-1=\frac{a}{c}$$

$$a=-c$$

Point $(3\,;3+c)$

$$3+c=\frac{a}{3+c}+1+c$$

$$3+c=\frac{-c}{3+c}+1+c$$

$$9+6c+c^2=-c+3+4c+c^2$$

$$3c=-6$$

$$c=-2$$

$$a=2$$

$$q=-1$$

$$f(x)=\frac{2}{x-2}-1$$

OR

$0 = x + c$
$\therefore \quad x = -c$

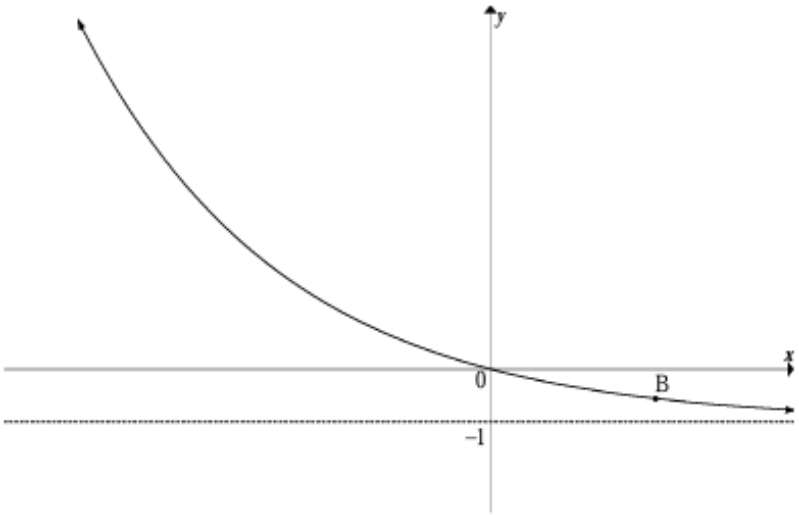
$q = 1 + c$

$f(x) = \frac{a}{x+c} + 1 + c$
$3 + c = \frac{a}{3+c} + 1 + c$
$2 = \frac{a}{3+c}$
$a = 6 + 2c$

$y = \frac{6+2c}{x+c} + 1 + c$
$c = \frac{6+2c}{0+c} + 1 + c$
$-1 = \frac{6+2c}{c}$
$-c = 6 + 2c$
$\therefore \quad c = -2$
$a = 2$
$q = -1$
$\therefore \quad f(x) = \frac{2}{x-2} - 1$

6.3	Translation of 1 unit down **OR/*OF*** Translation of 1 unit to the right

Paper E

4.1	$f(x) = a^x - 1$ $-\frac{5}{9} = a^2 - 1$ $a^2 = \frac{4}{9}$ $a = \frac{2}{3}$
4.2	$y > -1$ **OR/*OF*** $y \in (-1\,;\infty)$
4.3	
4.4	$\frac{19}{8} = \left(\frac{2}{3}\right)^x - 1$ $\frac{27}{8} = \left(\frac{2}{3}\right)^x$ $\left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^{-x}$ $\therefore x = -3$ $\therefore \mathrm{C}\left(-3\,;\frac{19}{8}\right)$ $\mathrm{C}'\left(\frac{19}{8}\,;-3\right)$

5.1	$p=-1$
5.2	$y=x-3$ $y=1-3$ $y=-2$
5.3	$1=\frac{a}{0-1}-2$ $3=\frac{a}{-1}$ $a=-3$
5.4	$\frac{-3}{x-1}-2=0$ $-3=2x-2$ $2x=-1$ $\therefore x=-\frac{1}{2}$ $x\in\left[-\frac{1}{2};1\right)$ **OR/*OF*** $-\frac{1}{2}\leq x<1$
5.5	f is reflected in the y-axis, then translated 2 units to the right **OR/*OF*** f is reflected in the x-axis, then translated 4 units down **OR/*OF*** f is reflected about the line $x=1$ **OR/*OF*** f is reflected about the line $y=-2$

6.1

$x = \dfrac{-4}{2(-1)} = 2$ **OR/*OF*** $f'(x) = -2x + 4 = 0$

$x = 2$

$y = -(2)^2 + 4(2) + 5$

$= 9$

$\therefore B(2\,;\,9)$

6.2

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or $x = -1$

$\therefore A(-1\,;\,0)$

Equation of AC:

$\dfrac{8-0}{3+1} = \dfrac{y-0}{x+1}$ **OR/*OF*** $m = \dfrac{8-0}{3+1} = 2$

$8x + 8 = 4y$ $y - 0 = 2(x+1)$

$2x + 2 = y$ $y = 2x + 2$

6.3

$EH = -x^2 + 4x + 5 - (2x + 2)$

$EH = -x^2 + 2x + 3$

Max EH at:

$-2x + 2 = 0$ **OR/*OF*** $x = \dfrac{-b}{2a} = \dfrac{-(-2)}{2(-1)}$

$x = 1$ $x = 1$

Max length of EH $= -(1)^2 + 2(1) + 3$

$= 4$

6.4

$k(x) = f(x+m)$

$= -x^2 - 2mx - m^2 + 4x + 4m + 5$

$k'(x) = -2x - 2m + 4$

$k'(x) = 2$

$-2x - 2m + 4 = 2$

$-2x = 2m - 2$

$x = 1 - m$

$k(1-m) = g(1-m)$

$-(1-m)^2 - 2m(1-m) - m^2 + 4(1-m) + 4m + 5 = 2(1-m) + 2$

$-1 + 2m - m^2 - 2m + 2m^2 - m^2 + 4 - 4m + 4m + 5 = 2 - 2m + 2$

$-2m - 4 = 0$

$\therefore m = -2$

OR

$k(x) = f(x+m) = g(x)$

$-x^2 - 2mx - m^2 + 4x + 4m + 5 = 2x + 2$

$0 = x^2 + (2m-2)x + m^2 - 4m - 3$

$\Delta = (2m-2)^2 - 4(1)(m^2 - 4m - 3)$

$\Delta = 4m^2 - 8m + 4 - 4m^2 + 16m + 12$

$\Delta = 8m + 16 = 0$

$\therefore m = -2$

OR/*OF*

$f(x) = -x^2 + 4x + 5$

$f'(x) = -2x + 4 = 2$

$-2x = -2$

$x = 1$

$f(1) = -(1)^2 + 4(1) + 5$

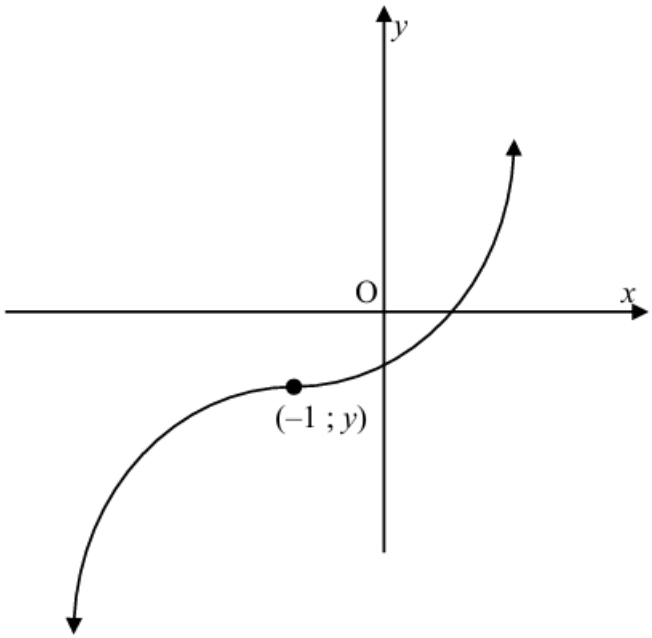
$\therefore y = 8$

$\therefore$ translate 2 units to the right

$\therefore m = -2$

Paper F

4.1	M(3 ; 4)
4.2	$f(x)=\frac{4}{x-3}+4$ $y=\frac{4}{0-3}+4=\frac{8}{3}$ $\therefore \mathrm{D}\left(0;\frac{8}{3}\right)$
4.3	M (3 ; 4) $y=x+t$ **OR/*OF*** $y=(x+p)+q$ $4=3+t$ $y=x-3+4$ $t=1$ $y=x+1$ $\therefore t=1$
4.4	$\frac{4}{x-3}+4=0$ $-4(x-3)=4$ $x-3=-1$ $x=2$ C(2 ; 0) $\therefore 2\le x<3$
4.5	$\frac{4}{x-3}+4=x+1$ $\frac{4}{x-3}=x-3$ $4=(x-3)^2$ $\pm 2=x-3$ $\therefore x=5$ or $x\neq 1$ $\therefore$ A (5 ; 6) **OR/*OF*** Point closest to the origin in $y=\frac{a}{x}$ is $(\sqrt{a};\sqrt{a})$ By translation: $\mathrm{A}(\sqrt{a}+3;\sqrt{a}+4)$ A(5 ; 6)
4.6	$h(x)=\frac{-4}{x+3}+4$ $=\frac{4}{-x-3}+4$ $\therefore$ Reflection in $y-$ axis. $\mathrm{A}'(-5;6)$ $\mathrm{AA}'=10$

5.1	$y = a(x+1)^2 + 4$ $-4 = a(-3+1)^2 + 4$ $-8 = 4a$ $-2 = a$ $y = -2(x+1)^2 + 4$ $y = -2x^2 - 4x + 2$
5.2	$k < -4$
5.3	

6.1	$(0\,;-3)\quad -3=p^0+q \qquad \text{OR} \qquad y=p^x-4$ $q=-4$ $(3\,;4)\quad 4=p^3-4$ $p^3=8$ $p=2$ $\therefore f(x)=2^x-4$
6.2	$y>-4$ **OR/*OF*** $y \in (-4\,;\infty)$
6.3	$g(x)=mx+c$ E $(-3\,;0)$ $m=\dfrac{4-0}{3-(-3)}=\dfrac{2}{3}$ $y=\dfrac{2}{3}x+c$ $0=\dfrac{2}{3}(-3)+c \qquad \text{OR} \qquad y-0=\dfrac{2}{3}(x+3)$ $\therefore c=2$ $y=\dfrac{2}{3}x+2 \qquad\qquad y=\dfrac{2}{3}x+2$ **OR/*OF*** For g^{-1}: $y=mx-3$ $3=m(4)-3$ $m=\dfrac{3}{2}$ $\therefore g^{-1}(x)=\dfrac{3}{2}x-3$ For g: $2x+6=3y$ $\dfrac{2}{3}x+2=y$ $\therefore y=\dfrac{2}{3}x+2$
6.4	$g(x)=\dfrac{2}{3}x+2$ $x=\dfrac{2}{3}y+2$ $g^{-1}(x)=\dfrac{3}{2}x-3$

Paper G

4.1	$p=-1$
4.2	$y=\frac{a}{x-1}$ $-3=\frac{a}{0-1}$ $a=3$ $y=x^2+bx-3$ $0=(1)^2+(1)b-3$ $b=2$
4.3	$y=x^2+2x-3$ axis of sym: $x=\frac{-b}{2a}$ $x=\frac{-2}{2(1)}$ $x=-1$ $y=(-1)^2+2(-1)-3=-4$ $C(-1;-4)$ **OR/OF** $\frac{dy}{dx}=0$ $2x+2=0$ $x=-1$ $y=(-1)^2+2(-1)-3=-4$ $C(-1;-4)$
4.4	$y\in[-4;\infty)$ or $y\geq -4$
4.5	$m=\tan 45^\circ=1$ $y=mx+c$ $-4=(1)(-1)+c$ $c=-3$ $y=x-3$
4.6	No, the line passes through C and D **OR/OF** No, a tangent through turning point C will have a gradient of 0

4.7 $f(m-x) = f[-(x-m)]$

f is reflected in the y-axis and translated 1 unit to the left and 4 units upwards.

Therefore: $m = -1$

$q = 4$

OR/OF

Substitute $x = 0$ and $q = 4$ for one x- intercept

$h(x) = (m-x)^2 + 2(m-x) - 3 + q$

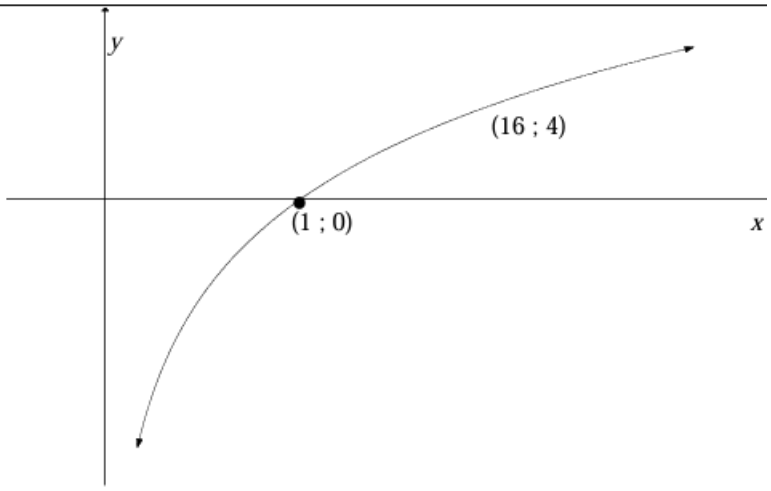
$h(0) = (m-0)^2 + 2(m-0) - 3 + 4$

$0 = m^2 + 2m + 1$

$0 = (m+1)^2$

$m = -1$

$q = 4$

5.1	$f(x) = k^x$ $16 = k^4$ $k = 2$
5.2	$f: \quad y = 2^x$ $f^{-1}: \quad x = 2^y$ $y = \log_2 x$
5.3	
5.4.1	$x \in (1\,;\infty)$ or $x > 1$
5.4.2	$0 < x \leq \frac{1}{2}$ or $x \in \left(0;\frac{1}{2}\right]$

5.5

$$2^x - 2^{-x} = \frac{15}{4}$$

$$2^x - \frac{1}{2^x} = \frac{15}{4}$$

$$2^{2x} - 1 = \frac{15}{4} \times 2^x$$

$$4.2^{2x} - 4 = 15 \times 2^x$$

$$4.2^{2x} - 15.2^x - 4 = 0$$

$$(4.2^x + 1)(2^x - 4) = 0$$

$4.2^x + 1 = 0$ or $2^x - 4 = 0$

$$2^x = \frac{-1}{4} \text{ or } 2^x = 2^2$$

N/A $x = 2$

OR/OF

$$2^x - 2^{-x} = \frac{15}{4}$$

$$2^x - \frac{1}{2^x} = \frac{15}{4}$$

Let $k = 2^x$

$$k^2 - 1 = \frac{15}{4} \times k$$

$$4.k^2 - 4 = 15 \times k$$

$$4.k^2 - 15.k - 4 = 0$$

$$(4.k + 1)(k - 4) = 0$$

$$k = \frac{-1}{4} \text{ or } k = 4$$

$$2^x = \frac{-1}{4} \text{ or } 2^x = 2^2$$

N/A $x = 2$

Paper H

4.1	$A(-4;3)$
4.2	For x-intercept, let $y=0$: $0=\frac{-9}{x+4}+3$ $-3=\frac{-9}{x+4}$ $x+4=3$ $x=-1$ $B(-1;0)$
4.3	For y-intercept, let $x=0$: $y=\frac{-9}{0+4}+3$ $y=\frac{3}{4}$ $C\left(0;\frac{3}{4}\right)$
4.4	Translate h 4 units to the right, and 3 units down.

4.5 The points of intersection between j and the axis of symmetry of j, i.e. between $y=\frac{-9}{x}$ and $y=-x$:

$$\frac{-9}{x}=-x$$
$$x^2=9$$
$$x=3 \quad \text{or} \quad x=-3$$
$$y=-3 \quad \text{or} \quad y=3$$

The points are $(3\,;-3)$ and $(-3\,;3)$

OR

Distance between origin and any point on the graph

$$=\sqrt{(0-x)^2+\left(0-\left(\frac{9}{x}\right)\right)^2}$$
$$=\sqrt{x^2+\frac{81}{x^2}}$$

For a minimum distance,

$$\frac{d}{dx}\left(x^2+\frac{81}{x^2}\right)=0$$
$$2x-162x^{-3}=0$$
$$2x^4=162$$
$$x^4=81$$
$$x=3 \quad \text{or} \quad x=-3$$

Therefore, the points on j closest to the origin are:
$(3\,;-3)$ and $(-3\,;3)$.

5.1 Substitute $\left(-\frac{1}{2}\ ;\ \frac{1}{2}\right)$ in $f(x)=a^x$:

$$a^{-\frac{1}{2}}=\left(\frac{1}{2}\right)$$

$$\frac{1}{\sqrt{a}}=\frac{1}{2}$$

$$\therefore \sqrt{a}=2$$

$$a=4$$

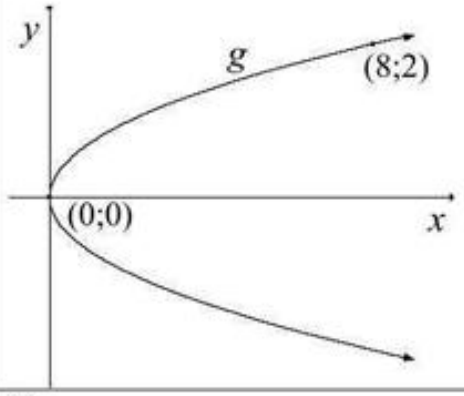
Substitute $\left(-\frac{1}{2}\ ;\ \frac{1}{2}\right)$ in $g(x)=bx^2$:

$$\frac{1}{2}=b\left(-\frac{1}{2}\right)^2$$

$$\frac{1}{2}=\frac{1}{4}b$$

$$\therefore b=2$$

5.2



5.3 No

.

5.4 $f(x)=4^x$ or $y=4^x$

Inverse: $x=4^y$

$y=\log_4 x$

domain of inverse = range of function

Therefore:

$y=\log_4 x,\ x\in\left[\frac{1}{4};16\right]$

5.5 $-\frac{1}{2}<x<0$

6.1

$$x^2+5x-6=0$$
$$(x+6)(x-1)=0$$
$$x=-6 \text{ or } x=1$$
$$A(-6;0); \quad B(1;0)$$

6.2

$$C(0;-6)$$
$$m=\frac{-6-0}{0+6}$$
$$=-1$$
$$y=-x-6$$

6.3

The x-values of the points of intersection are the roots of:

$$g(x)=f(x)+k$$
$$-x-6=x^2+5x-6+k$$
$$x^2+6x+k=0$$

If there are no points of intersection, the equation will have no real roots, i.e.:

$$b^2-4ac<0$$
$$6^2-4(1)(k)<0$$
$$-4k<-36$$
$$\therefore k>9$$

OR

f is translated upwards until there are no points of intersection between f and h.
Just before they do not intersect any more, h will be a tangent to f.
Calculating the value of k when h will be a tangent to f:

$$m_g=h'(x)$$
$$\therefore -1=2x+5$$
$$x=-3$$
$$g(-3)=-(-3)-6-k$$

$\therefore$ The contact point will be: $(-3;-3)$.

$$f(-3)=(-3)^2+5(-3)-6=-12$$

From -12 to -3 indicates an upward translation of 9 units.
$\therefore$ For g and h not to intersect, it means that f has to be translated upwards by **more than 9 units**.
$\therefore k>9$

Paper I

4.1.1

$f(x) = x^2 - 2x - 3$

$y-\text{int}(x=0):\ y=-3$

$x-\text{int}(y=0):\ x^2-2x-3=0$

$x=3 \ \ or \ \ x=-1$

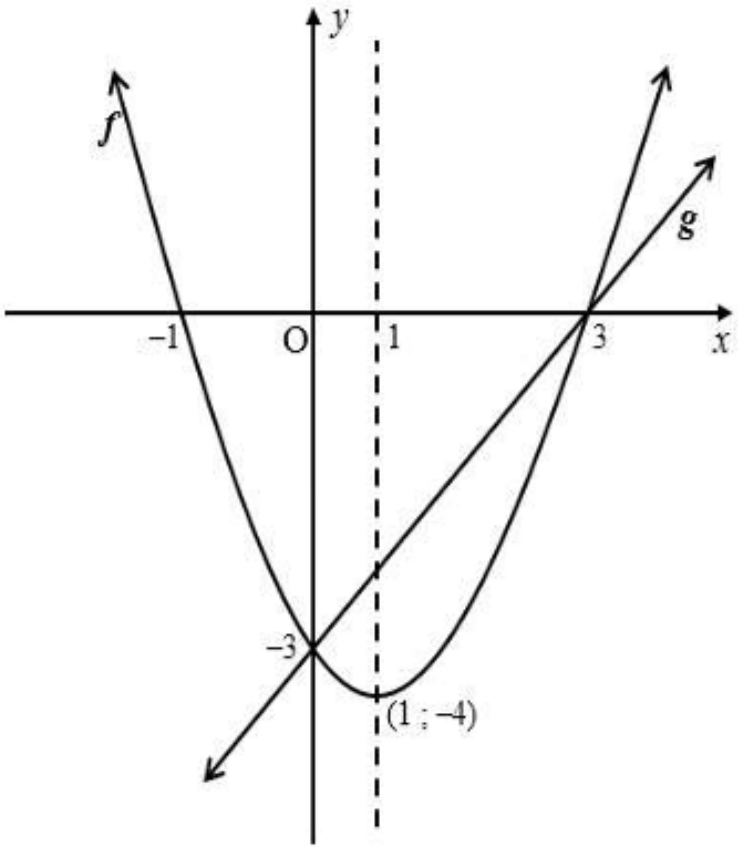
$x=-\dfrac{b}{2a}=\dfrac{-(-2)}{2(1)}=1$

$y=(1)^2-2(1)-3=-4 \ \Rightarrow \ TP\,is\,(1;-4)$

$g(x)=x-3$

$x-\text{int}(y=0):\ y=-3$

$y-\text{int}(x=0):\ x=3$



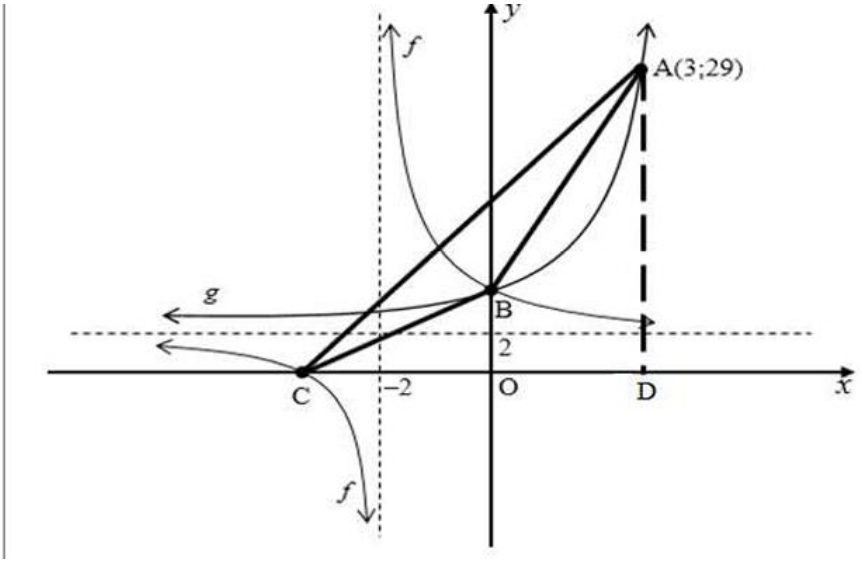
4.1.2 (a)	$x<0 \ \ or/of \ \ x>3$
4.1.2 (b)	$x\leq -1$
4.1.3	Range of h / *Waardeversameling (Terrein) van h* : $y \leq 0; y \in \mathbb{R}$

4.2

$$-x+1=4$$
$$-x=3$$
$$x=-3 \quad \Rightarrow \quad \text{A}(-3;0)$$

$$y=a(x+p)^2+q$$
$$y=a(x-1)^2+8$$
$$0=a(-3-1)^2+8$$
$$-8=16a$$
$$-\frac{1}{2}=a$$
$$\Rightarrow p(x)=-\frac{1}{2}(x-1)^2+8$$

5.1	B(0 ; 3)
5.2	Range of / *Waardeversameling van* g: $y > 2; y \in \mathbb{R}$
5.3	$g(x) = b^x + 2$ $29 = b^3 + 2$ $3^3 = b^3$ $\therefore\ b = 3$
5.4	$\frac{2}{x+2} + 2 = 0$ $2 + 2x + 4 = 0$ $2x = -6$ $x = -3$
5.5	$h(x) = 3^x + 2 - 2$ $= 3^x$ $h^{-1}(x):\ x = 3^y$ $\therefore\ y = \log_3 x$
5.6	original axis of symmetry: *oorspronklikesimmetrie-**as*** $\boldsymbol{y = x}$ $\boldsymbol{\therefore y = (x + 2) + 2}$ $\boldsymbol{y = x + 4}$
5.7	$-3 \leq x < -2$



$Area\ of\ \Delta ACD = \frac{1}{2}(6)(29) = 87\ units^2\ /\ eenhede^2$

$Area\ of\ \Delta BCO = \frac{1}{2}(3)(3) = 4,5\ units^2\ /\ eenhede^2$

$Area\ of\ Trap.BODA = \left(\frac{3+29}{2}\right)\times 3 = 48\ units^2\ /\ eenhede^2$

$\therefore Area\ \Delta ABC = 87 - (4,5+48) = 34,5\ units^2\ /\ eenhede^2$

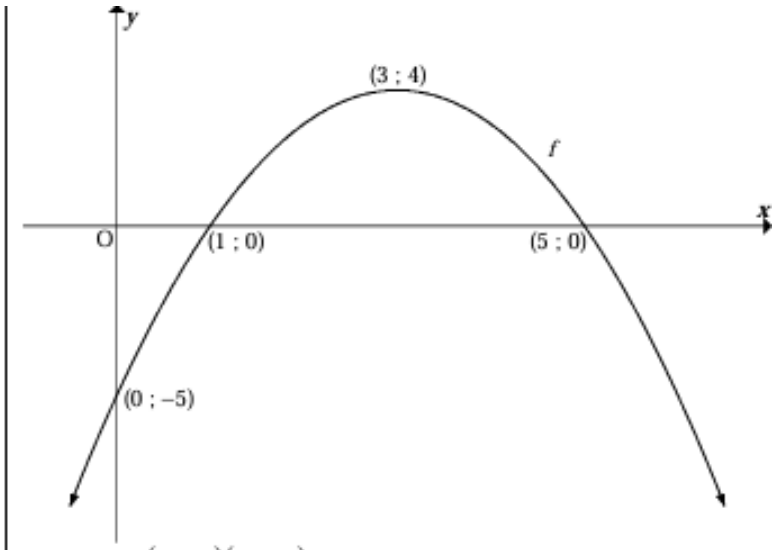
NOTE: There are many different alternative solutions to this question. Follow learner's solution and award marks accordingly.

Paper J

4.1	$U(1;0)$
4.2	$x=1$ $y=1$
4.3	$\frac{2}{x-1}+1=0$ $2=-x+1$ $x=-1$ $T(-1;0)$
4.4	$f(x)=\log_5 x$ $h:\ x=\log_5 y$ $y=5^x$
4.5	$y=0$
4.6	$V\left(\sqrt{2}+1;\sqrt{2}+1\right)$ $V(2{,}41;2{,}41)$ **OR / OF** $x=\frac{2}{x-1}+1$ $x^2-x=2+x-1$ $x^2-2x-1=0$ $x=\frac{2\pm\sqrt{4-4(1)(-1)}}{2}$ $=\frac{2\pm\sqrt{8}}{2}$ $=\frac{2\pm 2\sqrt{2}}{2}$ $=1\pm\sqrt{2}$ $V\left(1+\sqrt{2}\ ;1+\sqrt{2}\right)$ **OR / OF** $x-1=\frac{2}{x-1}$ $(x-1)^2=2$ $x=1\pm\sqrt{2}$ $V\left(1+\sqrt{2}\ ;1+\sqrt{2}\right)$
4.7	$T'(3;2)$

5.1.1	$C(0\,;-3)$
5.1.2	$f(x)=x^2-2x-3$ $(x-3)(x+1)=0$ $x=-1$ or $x=3$ AB = 3 – (–1) AB = 4 units
5.1.3	$x=\frac{2}{2(1)}$ or $2x-2=0$ or $x=\frac{-1+3}{2}$ $=1$ $y=(1)^2-2(1)-3$ $=-4$ $D(1\,;-4)$
5.1.4	$C(0\,;-3)$ $D(1\,;-4)$ Average gradient / *Gemiddelde gradiënt* $=\frac{-4+3}{1-0}$ or $\frac{-3+4}{0-1}$ $=-1$
5.1.5	OC = OB = 3 $O\hat{C}B=45°$ isosceles right angled triangle *Gelykbenige reghoekige driehoek* **OR / OF** $\tan\beta=m_g$ $\tan\beta=1$ $\beta=45°$ $O\hat{B}C=45°$ $O\hat{C}B=45°$
5.1.6	$-4<k<-3$ **OR** $(-4\,;-3)$
5.1.7	$f'(x)\,.\,f''(x)>0$ $(2x-2).2>0$ $2x-2>0$ $x>1$

5.2



$$f(x) = a(x-1)(x-5)$$
$$4 = a(3-1)(3-5)$$
$$4 = -4a$$
$$a = -1$$
$$f(x) = -x^2 + 6x - 5$$

FINANCE, GROWTH AND DECAY

Paper A

7.1

$$A = P(1+i)^n$$

$$A = 40\,000(1+7,8\%)^5$$

$$A = \text{R}58\,230,94$$

7.2

$$F = \frac{x\left[(1+i)^n - 1\right]}{i}$$

$$F = \frac{2\,300\left[\left(1+\frac{0,058}{4}\right)^{24} - 1\right]}{\frac{0,058}{4}} \times \left(1+\frac{0,058}{4}\right)$$

$$F = \text{R}66\ \ 411,60$$

OR/*OF*

$$F = \frac{2\,300\left[\left(1+\frac{0,058}{4}\right)^{25} - 1\right]}{\frac{0,058}{4}} - 2300$$

$$F = \text{R}66\,411,60$$

7.3.1

$$A = P(1+i)^n$$

$$A = 900\,000\left(1+\frac{0,068}{12}\right)^3$$

$$= \text{R}915\,386,86$$

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$915\,386,86 = \frac{10\,000\left[1-\left(1+\frac{0,068}{12}\right)^{-n}\right]}{\frac{0,068}{12}}$$

$$\left(1+\frac{0,068}{12}\right)^{-n} = 0,4812...$$

$$-n = \log_{1,005...} 0,4812...$$

$$n = 129,419... \text{ months}$$

$\therefore 132,419$ months since loan was granted

$\therefore 133$ months since loan was granted

7.3.2

$$P = \frac{10\,000\left[1-\left(1+\frac{0,068}{12}\right)^{-0,419\ldots}\right]}{\frac{0,068}{12}}$$

$$= \text{R}4\,173,55\ldots$$

$$\text{Final payment} = 4173,55\ldots\left(1+\frac{0,068}{12}\right)^{1} = R4\,197,21$$

OR/*OF*

$$\text{A} = 915\,386,86\left(1+\frac{0,068}{12}\right)^{129}$$

$$= \text{R}1\,897\,482,712$$

$$\text{F} = \frac{10\,000\left[\left(1+\frac{0,068}{12}\right)^{129}-1\right]}{\frac{0,068}{12}}$$

$$= \text{R}1\,893\,309,16$$

Balance after 129 months = R4 173,552

$$\text{Final payment} = 4\,173,552\left(1+\frac{0,068}{12}\right)^{1} = \text{R}4\,197,21$$

Paper B

6.1.1	$A = P(1+i)^n$ $19\,319{,}48 = 18\,500\left(1+\frac{r}{1200}\right)^6$ $\left(1+\frac{r}{1200}\right) = \sqrt[6]{1{,}04429\ldots}$ $\frac{r}{1200} = 0{,}00725\ldots$ $r = 8{,}7\%$
6.1.2	$1+\frac{i}{100} = \left(1+\frac{8{,}7}{1200}\right)^{12}$ $r = 9{,}06\%$
6.2.1	$A = P(1-in)$ $0 = 10\ 000(1-0{,}2n)$ $n = 5$
6.2.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $20\,000 = \frac{x\left[\left(1+\frac{8{,}7}{1200}\right)^{60} - 1\right]}{\frac{8{,}7}{1200}}$ $x = \text{R}267{,}26$

6.3

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$1\,600\,000 = \frac{20\,000\left[1-\left(1+\frac{0{,}112}{12}\right)^{-n}\right]}{\frac{0{,}112}{12}}$$

$$\frac{56}{75} = 1-\left(1+\frac{0{,}112}{12}\right)^{-n}$$

$$\left(1+\frac{0{,}112}{12}\right)^{-n} = \frac{19}{75}$$

$$-n = \log_{\left(1+\frac{0{,}112}{12}\right)}\left(\frac{19}{75}\right)$$

$$-n = -147{,}80$$

Tino will make 147 withdrawals of R20 000

Paper C

6.1

$$A = P(1+i)^n$$

$$13\,459 = 12\,000\left(1+\frac{m}{400}\right)^8$$

$$\left(1+\frac{m}{400}\right)^8 = 1{,}121\ldots$$

$$1+\frac{m}{400} = \sqrt[8]{1{,}121..}$$

$$\frac{m}{400} = 0{,}0144\ldots$$

$$\therefore m = 5{,}78\%$$

6.2

$$F = \frac{x\left[(1+i)^n - 1\right]}{i}$$

$$F = \frac{1\,000\left[\left(1+\frac{0{,}075}{12}\right)^{12} - 1\right]}{\frac{0{,}075}{12}}$$

$$= \text{R}12\,421{,}22$$

He won't be able to buy the computer because
R13 000 – R12 421,22 = R578,78
OR/OF
He won't be able to buy the computer because
R12 421,22 < R13 000

6.3.1

Loan amount = 85% × R250 000
= R212 500

OR/OF

Loan amount = R250 000 – (15% × R250 000)
= R212 500

6.3.2

$$A = 212\,500\left(1+\frac{0,13}{12}\right)^{5}$$

$$A = 224\,262,53$$

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$224\,262,53 = \frac{x\left[1-\left(1+\frac{0,13}{12}\right)^{-67}\right]}{\frac{0,13}{12}}$$

$$\therefore x = \text{R}4\,724,96$$

Paper D

7.1

$$(1+i)=\left(1+\frac{15}{1200}\right)^{12}$$

$$i=16{,}08\%$$

7.2

$$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$500\,000=\frac{11\,250\left[1-\left(1+\frac{0{,}06}{4}\right)^{-n}\right]}{\frac{0{,}06}{4}}$$

$$\frac{2}{3}=1-\left(1+\frac{0{,}06}{4}\right)^{-n}$$

$$(1{,}015)^{-n}=\frac{1}{3}$$

$$-n=\log_{1{,}015}\left(\frac{1}{3}\right)$$

$$n=73{,}788\ldots$$

$$\therefore n=73 \text{ withdrawals}$$

7.3

$$A=P(1+i)^n$$

$$=12\,000\left(1+\frac{0{,}095}{12}\right)^{12\times 4}$$

$$=\text{R } 17\,521{,}17895\ldots$$

$$F=\frac{x[(1+i)^n-1]}{i}$$

$$F=\frac{500\left[\left(1+\frac{0{,}095}{12}\right)^{24}-1\right]}{\frac{0{,}095}{12}}$$

$$=\text{R } 13\,158{,}64744\ldots$$

Total = 17 521,17895… + 13 158,64744…
= R30 679,83

Paper E

7.1	$A = P(1+i)^n$ $A = 5\,000\left(1+\frac{6,8}{400}\right)^{64}$ $= \text{R}14\,706,56$
7.2	$A = P(1-in)$ $\frac{1}{2}P = P(1-4i)$ $\frac{1}{2} = (1-4i)$ $i = \frac{1}{8}$ $\therefore r = 12,5\%$ **OR/*OF*** $r = \frac{50\%}{4}$ $r = 12,5\%$
7.3.1	Already paid: $2\,300,98 \times 60 = \text{R}\,138\,\,058,80$ Interest: $138\,058,80 - 100\,000 = \text{R}\,38\,058,80$

7.3.2

Balance after 2 years:

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$P = \frac{2\,300,98\left[1-\left(1+\frac{13,5}{1200}\right)^{-36}\right]}{\frac{13,5}{1200}}$$

$$= \text{R } 67\,804,94$$

R 67 804,94 – R20 000 = R 47 804,94

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$47\,804,94 = \frac{2\,300,98\left[1-\left(1+\frac{13,5}{1200}\right)^{-n}\right]}{\frac{13,5}{1200}}$$

$$0,766 = (1,01125)^{-n}$$

$$\log_{1,01125} 0,766 = -n$$

$$n = 23,83$$

$\therefore$ 12 months earlier

OR/*OF*

$$A = 100\,000\left(1+\frac{13,5}{1200}\right)^{12\times 2}$$

$$= \text{R}130\,799,12$$

$$F = \frac{2\,300,98\left[\left(1+\frac{13,5}{1200}\right)^{12\times 2}-1\right]}{\frac{13,5}{1200}}$$

$$= \text{R } 62\,993,92$$

Balance outstanding = A – F = 130 799,12 – 62 993,92

= R67 805,20

R 67 805,20 – R20 000 = R 47 805,20

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$47\,805,20 = \frac{2\,300,98\left[1-\left(1+\frac{13,5}{1200}\right)^{-n}\right]}{\frac{13,5}{1200}}$$

$$0,766... = (1,01125)^{-n}$$

$$\log_{1,01125} 0,766... = -n$$

$$n = 23,80$$

$\therefore$ 12 months' earlier

OR

$$100\ 000 = \frac{2\ 300{,}98\left[1-\left(1+\frac{13{,}5}{1200}\right)^{-n}\right]}{\frac{13{,}5}{1200}} + 20\ 000\left(1+\frac{13{,}5}{1200}\right)^{-24}$$

$$84\ 709{,}38 = \frac{2\ 300{,}98\left[1-\left(\frac{809}{800}\right)^{-n}\right]}{\frac{0{,}135}{12}}$$

$$0{,}4142 = 1-\left(\frac{809}{800}\right)^{-n}$$

$$\left(\frac{809}{800}\right)^{-n} = 0{,}5858$$

$$-n = \log_{\left(\frac{809}{800}\right)} 0{,}5858$$

$n = 47{,}80$ months after starting payments

$\therefore$ 12 months earlier

Paper F

6.1.1

$$A = 150\,000(1-0,2)^2$$
$$= \text{R}96\,000$$

6.1.2

$$150\,000(1-0,2)^n = 49\,152$$
$$(0,8)^n = \frac{1024}{3125}$$
$$n\log(0,8) = \log\frac{1024}{3125}$$
$$n = 5$$

The machine will need to be replaced at the beginning of 2020 / *Masjien moet aan die begin van 2020 vervang word*

OR / OF

$$150\,000(1-0,2)^n = 49\,152$$
$$(0,8)^n = \frac{1024}{3125}$$
$$n = \log_{0,8}\frac{1024}{3125}$$
$$n = 5$$

The machine will need to be replaced at the beginning of 2020 / *Masjien moet aan die begin van 2020 vervang word*

6.1.3

R280 000 – R49 152
=R230 848

$$230\,848 = \frac{x\left[\left(1+\frac{0{,}085}{4}\right)^{20}-1\right]}{\frac{0{,}085}{4}}$$

$$x = \text{R}9\,383{,}26$$

6.2

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$

$$= \frac{9\,000\left[1-\left(1+\frac{0{,}11}{12}\right)^{-180}\right]}{\frac{0{,}11}{12}}$$

$$= \text{R}791\,837{,}43$$

Lerato qualifies for a loan of R 791 000 under the given conditions / *Lerato kwalifiseer vir 'n lening van* R 791 000 *gegewe die kondisies*

Paper G

6.1

$$A = P(1+i)^n$$

$$13\,459 = 12\,000\left(1+\frac{m}{400}\right)^8$$

$$\left(1+\frac{m}{400}\right)^8 = 1{,}121\ldots$$

$$1+\frac{m}{400} = \sqrt[8]{1{,}121..}$$

$$\frac{m}{400} = 0{,}0144\ldots$$

$$\therefore m = 5{,}78\%$$

6.2

$$F = \frac{x\left[(1+i)^n - 1\right]}{i}$$

$$F = \frac{1\,000\left[\left(1+\frac{0{,}075}{12}\right)^{12} - 1\right]}{\frac{0{,}075}{12}}$$

$$= \text{R}12\,421{,}22$$

He won't be able to buy the computer because
R13 000 – R12 421,22 = R578,78
OR/OF
He won't be able to buy the computer because
R12 421,22 < R13 000

6.3.1

Loan amount = 85% × R250 000
= R212 500

OR/OF
Loan amount = R250 000 – (15% × R250 000)
= R212 500

6.3.2

$$A = 212\,500\left(1+\frac{0{,}13}{12}\right)^5$$

$$A = 224\,262{,}53$$

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$224\,262{,}53 = \frac{x\left[1-\left(1+\frac{0{,}13}{12}\right)^{-67}\right]}{\frac{0{,}13}{12}}$$

$$\therefore x = \text{R}4\,724{,}96$$

Paper H

6.1

Kuda : $A = P(1+in)$

$= 5\ 000(1+0{,}083\times 4)$

$= R6\ 660{,}00$

Final Answer : R6 660,00 + R266,40

$= R6\ 926{,}40$

OR/OF

Kuda : $A = P(1+in)\times 1{,}04$

$= 5\ 000(1+0{,}083\times 4)\times 1{,}04$

$= R6\ 926{,}40$

Thabo : $A = P(1+i)^n$

$= 5\ 000\left(1+\frac{0{,}081}{12}\right)^{12\times 4}$

$= R6\ 905{,}71$

Kuda will have a better investment

6.2.1

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$525\ 000 = \frac{6\ 000\left[1-\left(1+\frac{0{,}1}{12}\right)^{-n}\right]}{\frac{0{,}1}{12}}$$

$$\frac{35}{48} = 1-\left(1+\frac{0{,}1}{12}\right)^{-n}$$

$$-n\log\left(1+\frac{0{,}1}{12}\right) = \log\frac{13}{48}$$

$$-n = \frac{\log\frac{13}{48}}{\log\left(1+\frac{0{,}1}{12}\right)}$$

$$n = 157{,}40$$

$n = 158$ payments

OR

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$525\,000 = \frac{6\,000\left[1-\left(1+\frac{0{,}1}{12}\right)^{-12n}\right]}{\frac{0{,}1}{12}}$$

$$\frac{35}{48} = 1-\left(1+\frac{0{,}1}{12}\right)^{-12n}$$

$$-12n\log\left(1+\frac{0{,}1}{12}\right) = \log\frac{13}{48}$$

$$-12n = \frac{\log\frac{13}{48}}{\log\left(1+\frac{0{,}1}{12}\right)}$$

$$n = \frac{\log\frac{13}{48}}{\log\left(1+\frac{0{,}1}{12}\right)} \times \frac{1}{12}$$

$$n = 13{,}11686841$$

Number of payments = $13{,}11686841 \times 12 = 157{,}40$

n = 158 payments

6.2.2 Difference: R6 000 – R5 066,36 = R933,64

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$F = \frac{933{,}64\left[\left(1+\frac{0{,}1}{12}\right)^{108}-1\right]}{\frac{0{,}1}{12}}$$

$= R162\,503{,}51$

OR

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$F = \frac{6000\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$$

$$= \text{R}1\ 044\ 322,28$$

$$F = \frac{5\ 066,36\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$$

$$F = \text{R}881\ 818,77.....$$

Amount available for withdrawal
= R1 044 322,28 – R 881 818,77
= R162 503,51

OR/OF

Outstanding balance with monthly repayment of R5 066,35

$$= 525000\left(1+\frac{0,1}{12}\right)^{108} - \frac{5\ 066,36\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$$

= R404 666, 23

Outstanding balance with monthly repayment of R6 000

$$= 525000\left(1+\frac{0,1}{12}\right)^{108} - \frac{6\ 000\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$$

= R242 162,72

Amount available for withdrawal
R404 666,23 – R242 162,72 = R162 512,18

Paper I

7.1

$A = P(1+i)^n$

$\therefore 2x = x\left(1+\frac{i}{4}\right)^{24}$

$\therefore (1+\frac{i}{4})^{24} = 2$

$\therefore 1+\frac{i}{4} = \sqrt[24]{2}$

$\therefore \frac{i}{4} = 0{,}0293...$

$\therefore i = 0{,}1172...$

$\therefore$ Annual % interest rate = 11,72%

7.2.1

$A = P(1+i)^n$

$\therefore A = 10000(1+0{,}0079...)^5$

$\therefore A = 10000(1{,}0079...)^5$

$\therefore A = \text{R}10\,402{,}15$

7.2.2.

$P = \frac{x[1-(1+i)^{-n}]}{i}$

$\therefore \frac{450[1-(1,0079)^{-n}]}{0,0079} = 10402{,}15$

$\therefore [1-(1,0079)^{-n}] = 0{,}183$

$\therefore 1-0{,}183 = (1,0079)^{-n}$

$\therefore 0{,}8169 = (1,0079)^{-n}$

$\therefore -n = \log_{1,0079} 0{,}8169$

$\therefore -n = -25{,}63...$

$\therefore n = 26$

NOTE: Answer must be a natural number.

7.2.3

Balance of the loan after the 25^{th} payment:

= value of loan - value of annuity at that time

$= 10\,402{,}15(1{,}0079)^{25} - \frac{450[(1{,}0079)^{25}-1]}{0{,}0079}$

$= 12\,668{,}89 - 12\,386{,}53$

$= \text{R}282{,}36$

$$Pv = \frac{450\left[1-\left(1+\frac{0.095}{12}\right)^{-0,63\,151\,28}\right]}{\frac{0.095}{12}}$$

$$Pv = R282,36$$

Paper J

6.1

$A = P(1 + in)$
$100000 = 50000(1 + 0.085n)$
$2 = 1 + 0.085n$
$\therefore n = 11.7647\ldots$
$n = 11 \text{ years/10 months}$

6.2.1

$$A = P(1+i)^n$$
$$A = 250000\left(1+\frac{0.15}{12}\right)^2$$
$$A = R256\ 289.06$$

6.2.2

$$P_V = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$
$$256289.06 = \frac{x\left[1-\left(1+\frac{0.15}{12}\right)^{-46}\right]}{\frac{0.15}{12}}$$
$$x = \frac{256289.06 \times \frac{0.15}{12}}{\left[1-\left(1+\frac{0.15}{12}\right)^{-46}\right]}$$
$$x = R7359.79$$

6.3

$$x\left(1+\frac{0.12}{12}\right)^{84} + 2x\left(1+\frac{0.12}{12}\right)^{48} = R276558.75$$
$$x\left[\left(1+\frac{0.12}{12}\right)^{84} + 2\left(1+\frac{0.12}{12}\right)^{48}\right] = R276558.75$$
$$x = \frac{276558.75}{\left(1+\frac{0.12}{12}\right)^{84} + 2\left(1+\frac{0.12}{12}\right)^{48}}$$
$$x = \text{R50 000.00}$$

OR

$$x\left(1+\frac{0.12}{12}\right)^{36}=1.430768784x$$

$$\therefore 3.430768784x\left(1+\frac{0.12}{12}\right)^{48}=5.5311749x$$

$$\therefore 5.5311749x=276\ 558.75$$

$$\therefore x=R50\ 000$$

DIFFERENTIAL CALCULUS

Paper A

8.1

$$f(x)=\frac{1}{x}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{x-(x+h)}{x(x+h)}\times\frac{1}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$$

$$f'(x)=-\frac{1}{x^2}$$

OR/*OF*

OR/*OF*

$$f(x)=\frac{1}{x}$$

$$f(x+h)=\frac{1}{x+h}$$

$$f(x+h)-f(x)=-\frac{h}{x(x+h)}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$$

$$f'(x)=-\frac{1}{x^2}$$

8.2.1	$\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ $=\frac{d}{dx}\left(2x^3+\sqrt{2}.x^2\right)$ $=6x^2+2\sqrt{2}x$
8.2.2	$g(x)=\frac{3x^4-4x^2+6}{x^2}$ $g(x)=3x^2-4+6x^{-2}$ $g'(x)=6x-12x^{-3}$
9.1	$f(x)=ax^3+bx^2+cx-5$ $-5=a(0+1)^2(0-5)$ $-5=-5a$ $a=1$ $f(x)=(x+1)(x+1)(x-5)$ $f(x)=(x^2+2x+1)(x-5)$ $f(x)=x^3-3x^2-9x-5$ $\therefore b=-3$ and $c=-9$
9.2	$f(x)=x^3-3x^2-9x-5$ $f'(x)=3x^2-6x-9$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3$ or $x=-1$ Minimum value at $x=3$
9.3	$f''(x).f(x)>0$ Point of inflection: $x=1$ $x<1$; $x\neq-1$ or $x>5$

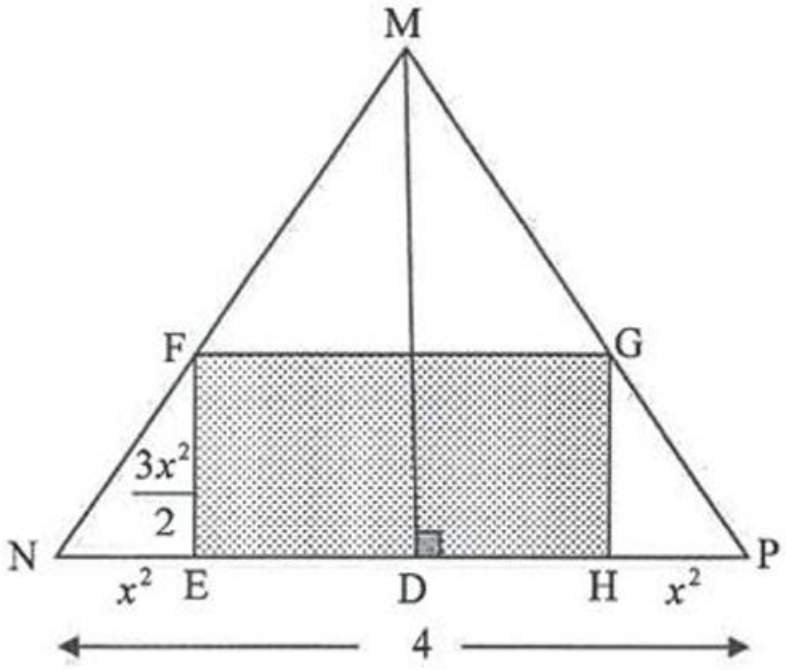
9.4 $-32 < -t < -5$

$5 < t < 32$

OR/*OF*

Shift up less than 32 units

$\therefore 5 < t < 32$



10.1

$$\frac{NE}{EF}=\frac{2}{3}=\frac{x^2}{b}$$

$$3x^2=2b$$

$$\therefore b=\frac{3x^2}{2}$$

$$EH=4-2x^2$$

$$\text{Area EFGH}=(4-2x^2)\left(\frac{3x^2}{2}\right)$$

$$A(x)=6x^2-3x^4$$

OR/OF

$$\text{In } \Delta DMP:\ \tan P=\frac{3}{2}$$

$$\text{In } \Delta HGP:\ \tan P=\frac{GH}{x^2}$$

$$\frac{GH}{x^2}=\frac{3}{2}$$

$$\therefore b=\frac{3x^2}{2}$$

$$EH=4-2x^2$$

$$\text{Area EFGH}=(4-2x^2)\left(\frac{3x^2}{2}\right)$$

$$A(x)=6x^2-3x^4$$

10.2

$$A(x)=6x^2-3x^4$$

$$A'(x)=12x-12x^3=0$$

$$12x(1-x^2)=0$$

$$\therefore x\neq 0 \text{ or } x=-1 \text{ or } x=1$$

$$\therefore \text{ max area: } A(1)=6(1)^2-3(1)^4=3 \text{ m}^2$$

Paper B

7.1

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-4(x+h)^2-\left(-4x^2\right)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-4x^2-8xh-4h^2+4x^2}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-8xh-4h^2}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{h(-8x-4h)}{h}$$

$$f'(x)=\lim_{h\to 0}(-8x-4h)$$

$$f'(x)=-8x$$

OR/*OF*

$$f(x+h)=-4(x+h)^2=-4x^2-8xh-4h^2$$

$$f(x+h)-f(x)=-4x^2-8xh-4h^2-(-4x^2)$$

$$=-8xh-4h^2$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

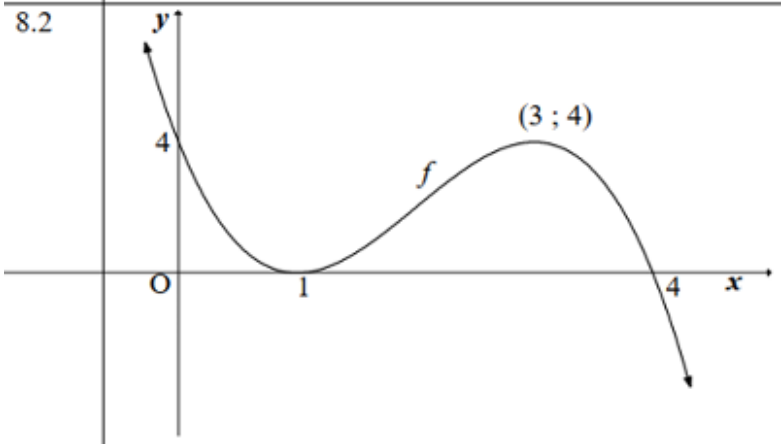
$$f'(x)=\lim_{h\to 0}\frac{-8xh-4h^2}{h}$$

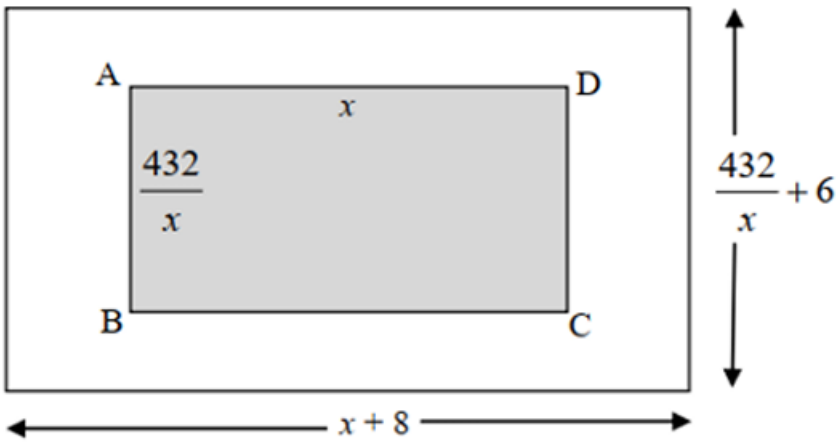
$$f'(x)=\lim_{h\to 0}\frac{h(-8x-4h)}{h}$$

$$f'(x)=\lim_{h\to 0}(-8x-4h)$$

$$f'(x)=-8x$$

7.2.1	$f(x) = 2x^3 - 3x$ $f'(x) = 6x^2 - 3$
7.2.2	$D_x\left[7.\sqrt[3]{x^2} + 2x^{-5}\right]$ $D_x\left[7.x^{\frac{2}{3}} + 2x^{-5}\right]$ $= \frac{14}{3}x^{-\frac{1}{3}} - 10x^{-6}$
7.3	$-6x^2 + 8 > 0$ $x^2 < \frac{8}{6}$ CV's: $x = -\frac{2}{\sqrt{3}}$ or $x = \frac{2}{\sqrt{3}}$ Positive for : $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$

8.1	$f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3 \text{ or } x = 1$ $f(3) = -(3)^3 + 6(3)^2 - 9(3) + 4 = 4$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) + 4 = 0$ $\therefore$ turning points are: (3 ; 4) and (1 ; 0)
8.2	
8.3	$0 < k < 4$ or/*of* $k \in (0;4)$
8.4	$f''(x) = -6x + 12 = 0$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$
	OR/*OF* Point of inflection: $x = \frac{3+1}{2}$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$
8.5	$\tan\theta = 3$ $\therefore \theta = 71{,}57°$



9.1

$$432 = xb$$

$$\therefore b = \frac{432}{x}$$

$$A(x) = (x+8)\left(\frac{432}{x}+6\right)$$

$$A(x) = 432 + 6x + \frac{3456}{x} + 480$$

$$A(x) = \frac{3456}{x} + 6x + 480$$

9.2

$$A(x) = 3456x^{-1} + 6x + 480$$

$$A'(x) = -\frac{3456}{x^2} + 6$$

$$-\frac{3456}{x^2} + 6 = 0$$

$$3456 = 6x^2$$

$$\therefore x = \sqrt{576} = 24\,\text{cm}$$

Paper C

8.1

$$f(x) = -2x + 3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-2(x+h) + 3 - (-2x+3)}{h}$$

$$= \lim_{h \to 0} \frac{-2h}{h}$$

$$= \lim_{h \to 0} (-2)$$

$$= -2$$

OR/*OF*

$$f(x) = -2x + 3$$

$$f(x+h) = -2(x+h) + 3$$

$$f(x+h) - f(x) = -2x - 2h + 3 + 2x - 3 = -2h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-2h}{h}$$

$$= \lim_{h \to 0} (-2)$$

$$= -2$$

8.2.1

$$g(x) = -3x^4 + 2x$$

$$g'(x) = -12x^3 + 2$$

8.2.2

$$y = \frac{2x^4 + 1}{x^2}$$

$$y = 2x^2 + x^{-2}$$

$$\frac{dy}{dx} = 4x - 2x^{-3}$$

9.1

$f(x) = x^3 - 8x^2 + 5x + 14$

$f'(x) = 3x^2 - 16x + 5 = 0$

$(3x - 1)(x - 5) = 0$

$x = \frac{1}{3}$ or $x = 5$

E(5 ; − 36)

9.2

$f''(x) = 6x - 16 < 0$

$x < \frac{8}{3}$

OR/*OF*

$x = \frac{\frac{1}{3} + 5}{2} = \frac{8}{3}$

$x < \frac{8}{3}$

9.3

x-intercepts of f: $(-1 ; 0)$ and $(7 ; 0)$

$-1 < x < 2$ or $\frac{8}{3} < x < 7$

9.4

$3x^2 - 16x + 5 = -11$

$3x^2 - 16x + 16 = 0$

$(x - 4)(3x - 4) = 0$

$x = 4$ or $x = \frac{4}{3}$

At $x = 4$: $(4)^3 - 8(4)^2 + 5(4) + 14 = -11(4) + t$

$t = 14$

At $x = \frac{4}{3}$: $\left(\frac{4}{3}\right)^3 - 8\left(\frac{4}{3}\right)^2 + 5\left(\frac{4}{3}\right) + 14 = -11\left(\frac{4}{3}\right) + t$

$t = \frac{634}{27} = 23\frac{13}{27} = 23,48$

$$\therefore 14 < t < \frac{634}{27}$$

OR/*OF*

$$x^3 - 8x^2 + 5x + 14 = -11x + t$$
$$x^3 - 8x^2 + 5x + 14 + 11x = t$$
$$x^3 - 8x^2 + 16x + 14 = t$$

$$3x^2 - 16x + 16 = 0$$
$$(x-4)(3x-4) = 0$$
$$x = 4 \quad \text{or} \quad x = \frac{4}{3}$$

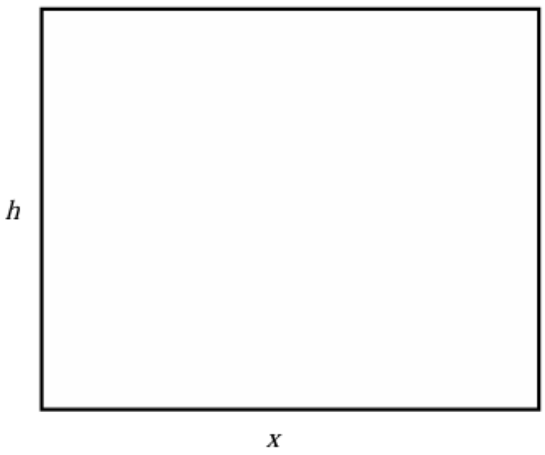
At $x = 4$: $\quad (4)^3 - 8(4)^2 + 5(4) + 14 = -11(4) + t$

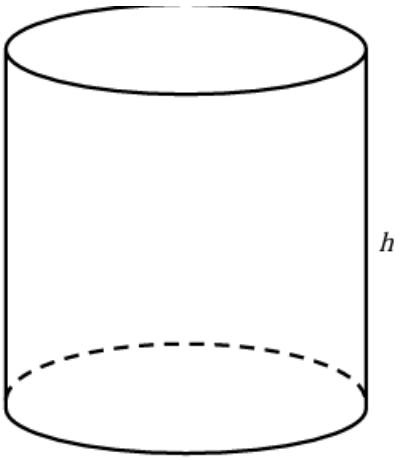
$$t = 14$$

At $x = \frac{4}{3}$: $\quad \left(\frac{4}{3}\right)^3 - 8\left(\frac{4}{3}\right)^2 + 5\left(\frac{4}{3}\right) + 14 = -11\left(\frac{4}{3}\right) + t$

$$t = \frac{634}{27} = 23\frac{13}{27} = 23,48$$

$$\therefore 14 < t < \frac{634}{27}$$





10.1

$2x+2h=50$

$h=25-x$

$2\pi r=x$

$r=\dfrac{x}{2\pi}$

$\mathrm{V}=\pi r^2 h$

$\mathrm{V}=\pi\left(\dfrac{x}{2\pi}\right)^2(25-x)$

$\mathrm{V}=\dfrac{\pi x^2}{4\pi^2}(25-x)$

$\mathrm{V}=\dfrac{25x^2}{4\pi}-\dfrac{x^3}{4\pi}$

10.2

$\mathrm{V}'(x)=\dfrac{50x}{4\pi}-\dfrac{3x^2}{4\pi}$

$\mathrm{V}'(x)=\dfrac{50x}{4\pi}-\dfrac{3x^2}{4\pi}=0$

$50x-3x^2=0$

$0=x(50-3x)$

$x\neq 0 \quad \text{or} \quad x=\dfrac{50}{3}$

$\therefore x=\dfrac{50}{3}=16,67$

Paper D

8.1

$f(x) = x^2 - 2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2 - (x^2 - 2)}{h}$

$f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$

$f'(x) = \lim_{h \to 0} \frac{h(2x + h)}{h}$

$f'(x) = \lim_{h \to 0} (2x + h)$

$\therefore f'(x) = 2x$

OR/*OF*

$f(x) = x^2 - 2$

$f(x+h) = (x+h)^2 - 2 = x^2 + 2xh + h^2 - 2$

$f(x+h) - f(x) = 2xh + h^2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$

$f'(x) = \lim_{h \to 0} \frac{h(2x + h)}{h}$

$f'(x) = \lim_{h \to 0} (2x + h)$

$\therefore f'(x) = 2x$

8.2.1

$\frac{d}{dx}\left[3x^2 - 4x\right]$

$= 6x - 4$

8.2.2

$g(x) = -2\sqrt{x}(x-1)^2$

$g(x) = -2x^{\frac{1}{2}}(x^2 - 2x + 1)$

$g(x) = -2x^{\frac{5}{2}} + 4x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$

$g'(x) = -5x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - x^{\frac{-1}{2}}$

8.3

$y = 4x - 14$ tangent

$\therefore 4x - 4 = 4$

$4x = 8$

$x = 2$

$\therefore y = 4(2) - 14$

at point $(2 ; -6)$

$g(2) = a(2)^2 + b(2) - 18 = -6$

$4a + 2b = 12 \quad ...(1)$

$g'(2) = 2a(2) + b = 4$

$4a + b = 4 \quad \ldots(2)$

$(1) - (2)$

$b = 8$

$4a + b = 4$

$4a + 8 = 4$

$4a = -4$

$a = -1$

OR/OF

$2x^2 - 4x - 6 = 4x - 14$

$2x^2 - 8x + 8 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$\therefore x = 2$

$\therefore y = 4(2) - 14$

at point $(2 ; -6)$

$g(2) = a(2)^2 + b(2) - 18 = -6$

$4a + 2b = 12 \quad ...(1)$

$g'(2) = 2a(2) + b = 4$

$4a + b = 4 \quad ...(2)$

$(1) - (2)$

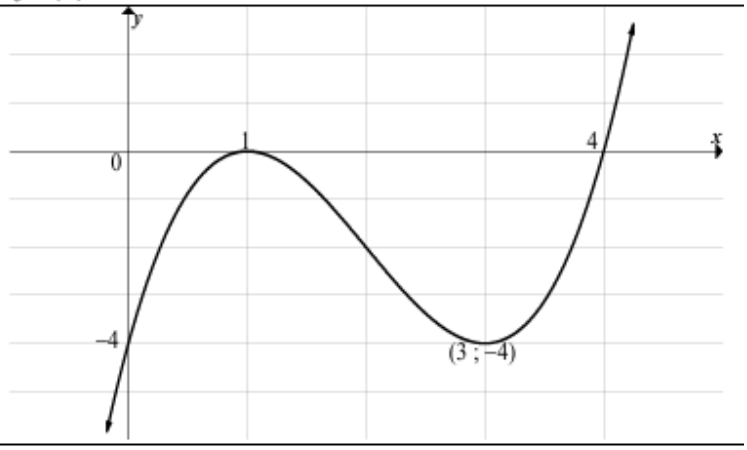
$b = 8$

$4a + b = 4$

$4a + 8 = 4$

$4a = -4$

$a = -1$

9.1	$f(x) = (x-4)(x^2-2x+1)$ $f(x) = (x-4)(x-1)^2$ $\therefore k = 1$ **OR/*OF*** $f(1) = 1-6+9-4 = 0$ $(x-1)$ is a factor $\therefore k = 1$ **OR/*OF*** $-4k^2 = -4$ $k^2 = 1$ $\therefore k = 1$
9.2	$3x^2 - 12x + 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $x = 3$ or $x = 1$ TP's: $(3\,;-4)$ and $(1\,;0)$
9.3	$f''(x) = 6x - 12$ $f''(-3) = -30$ $f''(x) < 0$, therefore concave down
9.4	
9.5	Distance $= (-6x^2 + 24x - 18) - (x^3 - 6x^2 + 9x - 4)$ $= -6x^2 + 24x - 18 - x^3 + 6x^2 - 9x + 4$ $= -x^3 + 15x - 14$ Max distance : $-3x^2 + 15 = 0$ $3x^2 = 15$ $x^2 = 5$ $x = \pm\sqrt{5}$, but $1 < x < 3$ $\therefore x = \sqrt{5}$ $d(\sqrt{5}) = 8{,}36$ max distance = 8,36

Paper E

8.1.1

$$\frac{d}{dx}\left[3x-5x^2\right]$$
$$=3-10x$$

8.1.2

$$g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$$
$$g(x)=2x^{-2}-x^{\frac{7}{3}}$$
$$g'(x)=-4x^{-3}-\frac{7}{3}x^{\frac{4}{3}}$$

8.2

$$f(x)=x^3-4x^2+2x+3$$
$$f'(x)=3x^2-8x+2$$
$$m=f'(2)=3(2)^2-8(2)+2$$
$$m=-2$$
$$y=f(2)=(2)^3-4(2)^2+2(2)+3$$
$$y=-1$$
$$y+1=-2(x-2)$$
$$y=-2x+3$$

8.3.1

$$f(x)=-6x^2$$
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{-6(x+h)^2+6x^2}{h}$$
$$=\lim_{h\to 0}\frac{-6x^2-12xh-6h^2+6x^2}{h}$$
$$=\lim_{h\to 0}\frac{-12xh-6h^2}{h}$$
$$=\lim_{h\to 0}\frac{h(-12x-6h)}{h}$$
$$=\lim_{h\to 0}(-12x-6h)$$
$$=-12x$$

OR

$$f(x) = -6x^2$$
$$f(x+h) = -6x^2 - 12xh - 6h^2$$
$$f(x+h) - f(x) = -6x^2 - 12xh - 6h^2 + 6x^2 = -12xh - 6h^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{-12xh - 6h^2}{h}$$
$$= \lim_{h\to 0} \frac{h(-12x - 6h)}{h}$$
$$= \lim_{h\to 0} (-12x - 6h)$$
$$= -12x$$

8.3.2

$x \geq 0$

OR/*OF*

$x \leq 0$

8.3.3

$$y = -6x^2$$
$$x = -6y^2$$
$$y^2 = \frac{-1}{6}x$$
$$y = \pm\sqrt{-\frac{1}{6}x}$$
$$\therefore y = -\sqrt{-\frac{1}{6}x} \quad ; \quad x \leq 0$$

9.1	$1 < x < \frac{5}{2}$
9.2	$(1\,;0)$ and $\left(\frac{5}{2}\,;0\right)$
9.3	$\frac{\frac{5}{2}+1}{2}$ $=\frac{7}{4}$ Concave up for $x > \frac{7}{4}$
9.4	$-9 < k < -8$

10.1	$-6t+18=0$ $18=6t$ $3=t$ $s'(3)=-3(3)^2+18(3)=27$
10.2	$-3t^2+18t=0$ $-3t(t-6)=0$ $t=0$ or $t=6$ $s(t)=at^3+bt^2$ $s'(t)=3at^2+2bt$ $3a=-3$ and $2b=18$ $a=-1$ $b=9$ $s(t)=-1t^3+9t^2$ $s(6)=-(6)^3+9(6)^2$ $s(6)=108$

Paper F

8.1

$$f(x) = 2x^2 - 5x + 3$$

$$f(x+h) = 2(x+h)^2 - 5(x+h) + 3$$

$$= 2(x^2 + 2xh + h^2) - 5x - 5h + 3$$

$$= 2x^2 + 4xh + 2h^2 - 5x - 5h + 3$$

$$f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 - 5x - 5h + 3 - (2x^2 - 5x + 3)$$

$$= 4xh + 2h^2 - 5h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h - 5)}{h}$$

$$= \lim_{h \to 0} (4x + 2h - 5)$$

$$= 4x - 5$$

8.2

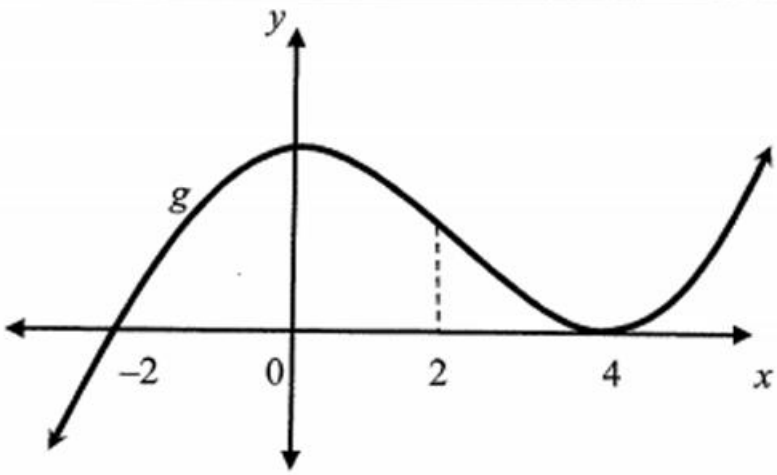
$$y = \frac{2x^2}{3\sqrt{x}} - \frac{2x^3 + 1}{x^3}$$

$$= \frac{2x^2}{3x^{\frac{1}{2}}} - 2 - \frac{1}{x^3}$$

$$= \frac{2}{3}x^{\frac{3}{2}} - 2 - x^{-3}$$

$$\frac{dy}{dx} = x^{\frac{1}{2}} + 3x^{-4}$$

9.1.1	$f(x) = -2x^3 + 5x^2 + 4x - 3$ $0 = (x-3)(-2x^2 - x + 1)$ $x - 3 = 0$ or $-2x^2 - x + 1 = 0$ $x = 3$ $2x^2 + x - 1 = 0$ $(3; 0)$ $(2x-1)(x+1) = 0$ $2x = 1$ or $x = -1$ $x = \frac{1}{2}$ $(-1; 0)$ $\left(\frac{1}{2}; 0\right)$
9.1.2	$f'(x) = -6x^2 + 10x + 4$ $0 = -6x^2 + 10x + 4$ $3x^2 - 5x - 2 = 0$ $(3x+1)(x-2) = 0$ or $x = \frac{-10 \pm \sqrt{10^2 - 4(-6)(4)}}{2(-6)}$ $3x = -1$ or $x = 2$ $x = -\frac{1}{3}$
9.1.3	$f''(x) = -12x + 10$ $0 = -12x + 10$ $12x = 10$ $x = \frac{10}{12} = \frac{5}{6}$ $\therefore x < \frac{5}{6}$

9.2.1	
9.2.2	$x < -2$ or $x > 2; x \neq 4$

10.1	$N(t) = t^3 - 12t^2 + 36t + 8$ $N(0) = 8$ $\therefore$ 8 people
10.2	$N'(t) = 3t^2 - 24t + 36$ increasing $N'(t) \geq 0$ $3t^2 - 24t + 36 \geq 0$ $t^2 - 8t + 12 \geq 0$ $(t - 6)(t - 2) \geq 0$ $t \geq 6$ or $t \leq 2$ $\therefore$ for first 2 hours after opening or 6 hours after opening until closing time
10.3	Minimum turning point at $t = 6$ hours after opening

Paper G

7.1

$$f(x+h) = (x+h)^2 - 5 = (x^2 + 2xh + h^2) - 5$$
$$= x^2 + 2xh + h^2 - 5$$
$$f(x+h) - f(x) = x^2 + 2xh + h^2 - 5 - \left(x^2 - 5\right)$$
$$= 2xh + h^2$$
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2}{h}$$
$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$
$$= \lim_{h\to 0} (2x+h)$$
$$= 2x$$

OR/OF

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 5 - \left(x^2 - 5\right)}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2}{h}$$
$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$
$$= \lim_{h\to 0} (2x+h)$$
$$= 2x$$

7.2

$$g(x) = 5x^2 - \frac{2x}{x^3}$$
$$= 5x^2 - 2x^{-2}$$
$$g'(x) = 10x + 4x^{-3}$$
$$= 10x + \frac{4}{x^3}$$

7.3

$h(x) = ax^2,\ x > 0$

$h^{-1}:\ x = ay^2 \quad y > 0$

$$y = \sqrt{\frac{x}{a}}$$

$$h^{-1}(8) = \sqrt{\frac{8}{a}}$$

$$h'(x) = 2ax$$

$$h'(4) = 2a(4)$$

$$= 8a$$

$$\sqrt{\frac{8}{a}} = 8a$$

$$64a^2 = \frac{8}{a}$$

$$a^3 = \frac{1}{8}$$

$$a = \frac{1}{2}$$

8.1

$$f'(x) = 0$$

$$6x^2 - 10x + 4 = 0$$

$$3x^2 - 5x + 2 = 0$$

$$(3x - 2)(x - 1) = 0$$

$$x = \frac{2}{3} \quad \text{or} \quad x = 1$$

$$y = 2\left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) \qquad y = 2(1)^3 - 5(1)^2 + 4(1)$$

$$y = \frac{28}{27} \quad \text{or} \quad y = 1$$

Turning points are $\left(\frac{2}{3}; \frac{28}{27}\right)$ and $(1;1)$

8.2

$$2x^3 - 5x^2 + 4x = 0$$
$$x(2x^2 - 5x + 4) = 0$$
$$x = 0 \quad \text{or} \quad x = \frac{5 \pm \sqrt{25 - 4(2)(4)}}{4}$$
$$= \frac{5 \pm \sqrt{-7}}{4}$$

No real roots / *Geen reële wortels*

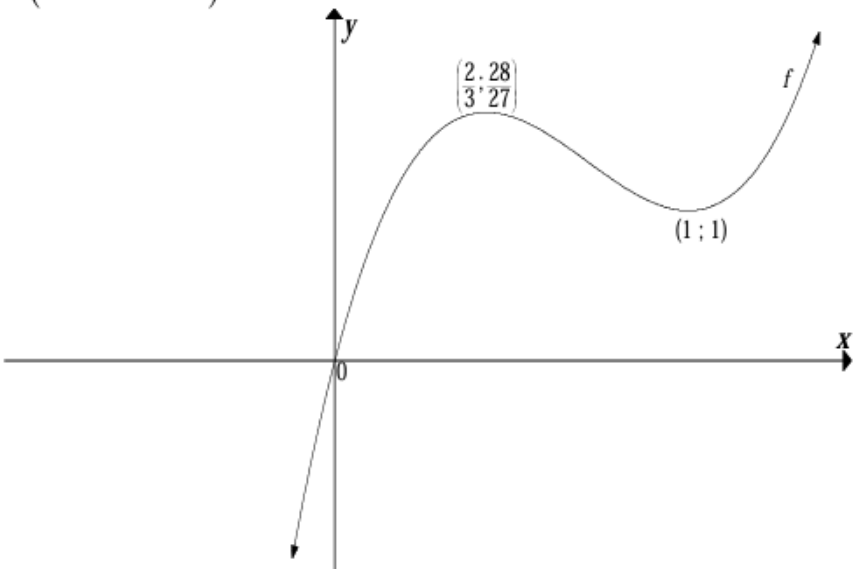
OR / OF

$$2x^3 - 5x^2 + 4x = 0$$
$$x(2x^2 - 5x + 4) = 0$$
$$x = 0 \quad \text{or} \quad b^2 - 4ac = 25 - 4(2)(4)$$
$$= -7 < 0$$

No real roots / *Geen reële wortels*

8.3

$$f(x) = 2x^3 - 5x^2 + 4x$$
$$x(2x^2 - 5x + 4) = 0$$



8.4

$$f(x) = 2x^3 - 5x^2 + 4x$$
$$f'(x) = 6x^2 - 10x + 4$$
$$f''(x) = 12x - 10$$
$$f''(x) > 0$$
$$12x - 10 > 0$$
$$x > \frac{5}{6}$$

OR

Point of inflection: $x = -\frac{b}{3a}$

$$x = -\frac{(-5)}{3(2)}$$

$$x = \frac{5}{6}$$

The function is concave up for $x > \frac{5}{6}$ since $a > 0$

OR

Point of inflection: $x = \frac{\frac{2}{3}+1}{2}$

$$x = \frac{5}{6}$$

The function is concave up for $x > \frac{5}{6}$ since $a > 0$

9.

Length of one side of the square / *lengte van sy van vierkant*

$$= \frac{x}{4}$$

Length of the rectangle / *lengte van die reghoek*:

$$2l + x + \frac{x}{4} = 6$$

$$l = \frac{6 - \frac{5x}{4}}{2}$$

$$= \frac{24 - 5x}{8}$$

$$A = \left(\frac{x}{4}\right)^2 + \frac{x}{4}\left(\frac{24 - 5x}{8}\right)$$

$$= \frac{x^2}{16} + \frac{24x - 5x^2}{32}$$

$$= \frac{24x - 3x^2}{32}$$

$$A = \frac{24x - 3x^2}{32}.$$

For minimum area / *Vir min imum oppervlaktte* $\frac{dA}{dx} = 0$

$$\frac{dA}{dx} = \frac{24 - 6x}{32}$$

$$6x = 24$$

$$x = 4$$

Paper H

8.1

$$f(x+h) = (x+h)^2 - 3(x+h)$$
$$= x^2 + 2xh + h^2 - 3x - 3h$$
$$f(x+h) - f(x) = x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)$$
$$= 2xh + h^2 - 3h$$
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h}$$
$$= \lim_{h\to 0} \frac{h(2x + h - 3)}{h}$$
$$= \lim_{h\to 0} (2x + h - 3)$$
$$= 2x - 3$$

OR/*OF*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$
$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h}$$
$$= \lim_{h\to 0} \frac{h(2x + h - 3)}{h}$$
$$= \lim_{h\to 0} (2x + h - 3)$$
$$= 2x - 3$$

8.2.1

$$y = \left(x^2 - \frac{1}{x^2}\right)^2$$
$$y = x^4 - 2 + \frac{1}{x^4}$$
$$= x^4 - 2 + x^{-4}$$
$$\frac{dy}{dx} = 4x^3 - 4x^{-5}$$

OR

By using the chain rule (which is not part of CAPS):

$$y = \left(x^2 - x^{-2}\right)^2$$

$$\frac{dy}{dx} = 2\left(x^2 - x^{-2}\right)\left(2x + 2x^{-3}\right)$$

$$= 2\left(2x^3 + 2x^{-1} - 2x^{-1} - 2x^{-5}\right)$$

$$= 2\left(2x^3 - 2x^{-5}\right)$$

$$= 4x^3 - 4x^{-5}$$

8.2.2

$$D_x\left[\frac{(x-1)(x^2+x+1)}{x-1}\right]$$

$$= D_x\left[x^2 + x + 1\right]$$

$$= 2x + 1$$

OR/*OF*

By using the quotient rule (with is not part of CAPS):

$$D_x\left[\frac{x^3-1}{x-1}\right]$$

$$= \frac{3x^2(x-1) - \left(x^3 - 1\right)}{(x-1)^2}$$

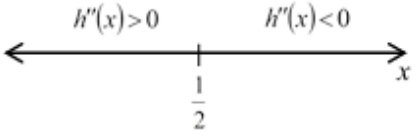
9.1	Substitute Q(2; 10) into $h(x) = -x^3 + ax^2 + bx$ $-2^3 + a(2^2) + b(2) = 10$ $-8 + 4a + 2b = 10$ $2a + b = 9$ .....line 1 $h'(x) = -3x^2 + 2ax + b$ At Q : $h'(2) = 0$ $-3(2)^2 + 2a(2) + b = 0$ $-12 + 4a + b = 0$ $4a + b = 12$ .....line 2 line 2 – line 1 : $2a = 3$ $a = \frac{3}{2}$ Substitute in line 1 : $b = 6$
9.2	$f(-1) = -(-1)^3 + \frac{3}{2}(-1)^2 + 6(-1)$ $= -3,5$ Average gradient/*Gemiddelde gradiënt* $= \frac{f(x_Q) - f(x_P)}{x_Q - x_P}$ Average gradient/ *Gemiddelde gradiënt* $= \frac{10-(-3,5)}{2-(-1)}$ $= 4,5$

9.3

$h'(x) = -3x^2 + 3x + 6$

$h''(x) = -6x + 3$

$= -3(2x - 1)$



For $x < \frac{1}{2}$, h is concave up and for $x > \frac{1}{2}$, h is concave down

Vir $x < \frac{1}{2}$, *is* h *konkaaf na bo en vir* $x > \frac{1}{2}$, *is* h *konkaaf na onder*

$\therefore$ concavity changes at $x = \frac{1}{2}$/

$\therefore$ *konkwiteit verander by* $x = \frac{1}{2}$

9.4

The graph of h has a point of inflection at $x = \frac{1}{2}$ /

Die grafiek van h het 'n buigpunt by $x = \frac{1}{2}$.

OR/*OF*

The graph of h changes from concave up to concave down at $x = \frac{1}{2}$ / *Die grafiek van h verander by* $x = \frac{1}{2}$ *van konkaaf op na konkaaf af*

9.5

Gradient of g is -12/*Gradiënt van g is –12*

Gradient of tangent is/*Gradiënt van die raaklyn is*:

$h'(x) = -3x^2 + 3x + 6$

$h'(x) = -12$

$-3x^2 + 3x + 6 = -12$

$3x^2 - 3x + 18 = 0$

$x^2 - x + 6 = 0$

$(x - 3)(x + 2) = 0$

$x = -2$ only

10.1

$$\frac{h}{r} = \tan 60^0$$

$$r = \frac{h}{\tan 60^0}$$

$$\therefore r = \frac{h}{\sqrt{3}}$$

10.2

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$$

$$= \frac{1}{9}\pi h^3$$

$$\frac{dV}{dh} = \frac{1}{3}\pi h^2$$

$$\left.\frac{dV}{dh}\right|_{h=9} = \frac{1}{3}\pi (9)^2$$

$$= 27\pi \quad \text{or} \quad 84{,}82\ \text{cm}^3/\text{cm}$$

Paper I

7.1

$$f(x)=x^2+x$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2+(x+h)-(x^2+x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{x^2+2xh+h^2+x+h-x^2-x}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2+h}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h+1)}{h}$$

$$\therefore f'(x)=2x+1$$

OR/*OF*

$$f(x)=x^2+x$$

$$f(x+h)=(x+h)^2+(x+h)=x^2+2xh+h^2+x+h$$

$$f(x+h)-f(x)=x^2+2xh+h^2+x+h-x^2-x$$

$$=2xh+h^2+h$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2+h}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h+1)}{h}$$

$$\therefore f'(x)=2x+1$$

7.2

$$f(x)=2x^5-3x^4+8x$$

$$f'(x)=10x^4-12x^3+8$$

7.3

$$g(x)=ax^3+3x^2+bx+c$$

$$g'(x)=3ax^2+6x+b$$

$$g''(x)=6ax+6$$

$$g''(-1)=6a(-1)+6=0$$

$$\therefore a=1$$

For concave up $g''(x)>0$

$$6x+6>0$$

$$x>-1$$

OR/OF

Min gradient at $(-1\ ;-7)$ implies:

at $x=-1$ - point of inflection
and g will be positive cubic
hence $a>0$

Since g is concave up

$x>-1$

OR/OF



Since g is concave up

$x>-1$



Since g is concave up

$x>-1$

OR/OF

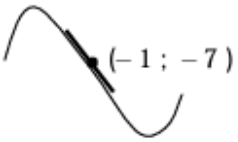
Min gradient at $(-1\ ;-7)$ implies:

at $x=-1$ - point of inflection
and g will be positive cubic
hence $a>0$

Since g is concave up

$x>-1$

OR/OF



Since g is concave up

$x>-1$



Since g is concave up

$x>-1$

OR/OF

Min gradient at $(-1\ ;-7)$ implies:

at $x = -1$ - point of inflection

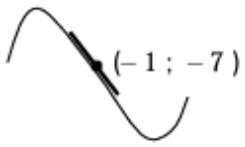
and g will be positive cubic

hence $a > 0$

Since g is concave up

$x > -1$

OR/OF



Since g is concave up

$x > -1$



$(-1\ ;\ y)$

Since g is concave up

$x > -1$

8.1

$f'(x) = mx^2 + nx + k$

$f'(x) = m\left(x + \frac{1}{3}\right)(x-1)$

$1 = m\left(0 + \frac{1}{3}\right)(0-1)$

$1 = -\frac{1}{3}m$

$\therefore m = -3$

$f'(x) = -3\left(x + \frac{1}{3}\right)(x-1)$

$f'(x) = -3\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$

$f'(x) = -3x^2 + 2x + 1$

$\therefore n = 2$

$\therefore k = 1$

OR/OF

$k = 1$

$0 = m + n + 1 \quad \text{and} \quad \frac{1}{9}m - \frac{1}{3}n + 1 = 0$

$m + n = -1 \qquad (1)$

$m - 3n = -9 \qquad (2)$

(1) – (2)

$4n = 8$

$\therefore n = 2$

$m + 2 = -1$

$\therefore m = -3$

8.2.1

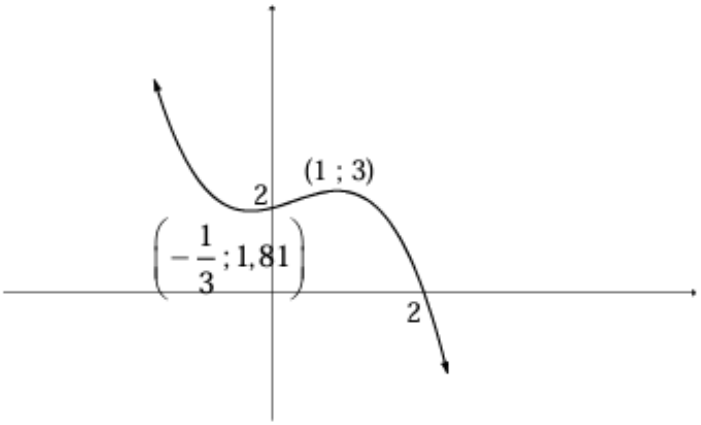
$f(x) = -x^3 + x^2 + x + 2$

$f\left(-\frac{1}{3}\right) = \frac{49}{27} = 1,81$

$\text{T.P}\left(-\frac{1}{3}; \frac{49}{27}\right)$

$f(1) = 3$

$\text{T.P}(1; 3)$

8.2.2	$f(x)=-x^3+x^2+x+2$ $-x^3+x^2+x+2=0$ $(x-2)(-x^2-x-1)=0$ $x=2$ or no solution 
8.3.1	$a=\frac{-\frac{1}{3}+1}{2}$ $=\frac{1}{3}$ **OR/OF** $f'(x)=-3x^2+2x+1$ $f''(x)=-6x+2$ $f''(a)=-6a+2=0$ $-6a=-2$ $a=\frac{1}{3}$
8.3.2	$b<\frac{4}{3}$ units

9.1

Any point on f : $(x\,;\,x^2)$

$$\text{distance} = \sqrt{(x-10)^2 + (x^2-2)^2}$$

$$= \sqrt{x^2 - 20x + 100 + x^4 - 4x^2 + 4}$$

$$= \sqrt{x^4 - 3x^2 - 20x + 104}$$

For min distance

$$\frac{d}{dx}\left(x^4 - 3x^2 - 20x + 104\right) = 0$$

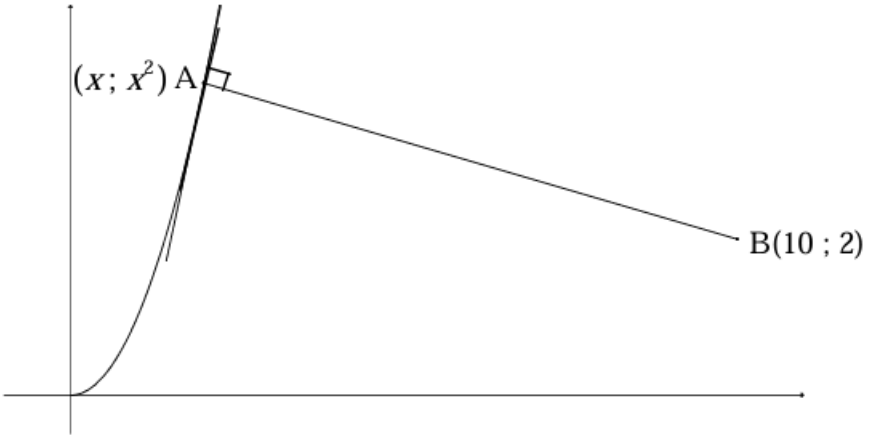
$$4x^3 - 6x - 20 = 0$$

$$(x-2)(4x^2 + 8x + 10) = 0$$

$$\Delta = 8^2 - 4(4)(10) = -96 \quad \therefore \text{no roots}$$

$$\therefore x = 2$$

$$d = \sqrt{2^4 - 3(2)^2 - 20(2) + 104} = 2\sqrt{17} = 8{,}25$$



9.2

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x^2 - 2}{x - 10}$$

$$\therefore m_{\tan g} = -\frac{x-10}{x^2-2}$$

$$f'(x) = 2x$$

$$\therefore 2x = -\frac{x-10}{x^2-2}$$

$$-2x^3 + 4x = x - 10$$

$$2x^3 - 3x - 10 = 0$$

$$x = 2$$

$$y = (2)^2 = 4$$

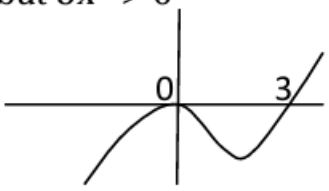
$$\therefore AB = \sqrt{(2-10)^2 + (4-2)^2}$$

$$= 2\sqrt{17} = 8,25$$

Paper J

7.1	$f(x) = 4 - 7x$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{4 - 7(x+h) - (4 - 7x)}{h}$ $= \lim_{h \to 0} \frac{h(-7)}{h}$ $= -7$
7.2	$y = 4x^8 + \sqrt{x^3}$ $= 4x^8 + x^{\frac{3}{2}}$ $\frac{dy}{dx} = 32x^7 + \frac{3}{2}x^{\frac{1}{2}}$
7.3.1	$y = ax^2 + a$ $\frac{dy}{dx} = 2ax + 0$ $\frac{dy}{dx} = 2ax$
7.3.2	$y = ax^2 + a$ $\frac{dy}{da} = x^2 + 1$

7.4	Substitute (2 ; b) in $y = x + \frac{12}{x}$ $b = 2 + \frac{12}{2}$ $b = 8$ $m_{\text{tangent}} = \frac{dy}{dx}$ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ $m_{\text{tangent}} = 1 - \frac{12}{2^2} = -2$ $m_{\text{perp}} = \frac{1}{2}$ Equation of perpendicular line: $y - y_1 = m(x - x_1)$ OR $y = mx + c$ $y - 8 = \frac{1}{2}(x - 2)$ $8 = \frac{1}{2}(2) + c$ $y = \frac{1}{2}x + 7$ $c = 7$ $y = \frac{1}{2}x + 7$
8.1	36cm
8.2	$\therefore t = 6$ $(-2t^2 + 3t - 6)$ have no real roots Insect reaches the floor only once.
8.3	$h(t) = -2t^3 + 15t^2 - 24t + 36$ $h'(t) = -6t^2 + 30t - 24$ $-6t^2 + 30t - 24 = 0$ $t^2 - 5t + 4 = 0$ $(t - 4)(t - 1) = 0$ $t = 4$ or $t = 1$ Only $t = 4$ because maximum value required $h = -2(4)^3 + 15(4)^2 - 24(4) + 36 = 52\ cm$

9.1	$f'(x) = 9x^2$ $3x^3 = 9x^2$ $3x^3 - 9x^2 = 0$ $3x^2(x-3) = 0$ $x = 0$ or $x = 3$
9.2.1	For f and f'
9.2.2	The point $(0\ ;\ 0)$ is : A point of inflection of f A turning point of f'
9.3	$f''(x) = 18x$ Distance $= f''(1) - f'(1)$ $= 18(1) - 9(1)^2$ $= 9$
9.4	$3x^3 - 9x^2 < 0$ $3x^2(x-3) < 0$ but $3x^2 > 0$  $\therefore x - 3 < 0$ $\therefore x < 3\ ,\ x \neq 0$

PROBABILITY AND COUNTING PRINCIPLES

Paper A

	JUICE	ENERGY DRINKS	TOTAL
Females	a	**72**	**120**
Males	36	54	**90**
Total	**84**	**126**	210

11.1.1

$$P(\text{Male and Prefer juice}) = P(\text{Male}) \times P(\text{Prefer juice})$$

$$\frac{36}{210} = \frac{90}{210} \times \frac{e}{210}$$

$$e = 84$$

11.1.2

$$P(\text{Female and like energy drink}) = \frac{b}{210}$$

$$d = 210 - 84 = 126$$

$$b = 126 - 54 = 72$$

$$P(\text{Female and like energy drink}) = \frac{72}{210} = \frac{12}{35} = 0,34$$

OR/*OF*

$$c = 120$$

$$b = 72$$

$$P(\text{Female and like energy drink}) = \frac{72}{210} = \frac{12}{35} = 0,34$$

11.2

$$\frac{3}{4}(3x)+\frac{1}{4}x=\frac{7}{12}$$

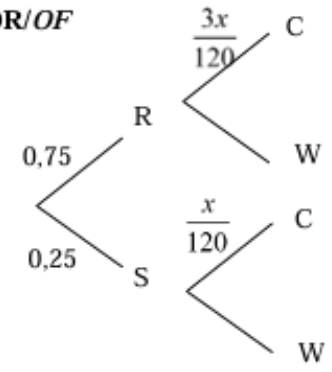
$$27x+3x=7$$

$$30x=7$$

$$x=\frac{7}{30}$$

$\therefore$ 28 cups

OR/*OF*



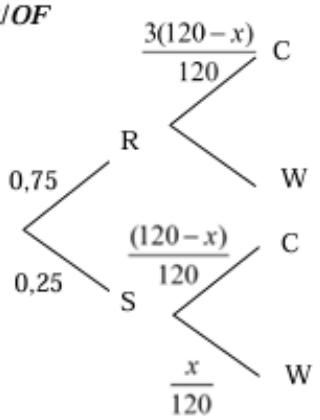
$$\frac{75}{100}\times\left(\frac{3x}{120}\right)+\frac{25}{100}\left(\frac{x}{120}\right)=\frac{7}{12}$$

$$\frac{3}{160}x+\frac{1}{480}x=\frac{7}{12}$$

$$10x=280$$

$x=28$ cups of coffee

OR/*OF*



$$\frac{75}{100}\times\left(\frac{3(120-x)}{120}\right)+\frac{25}{100}\left(\frac{120-x}{120}\right)=\frac{7}{12}$$

$$270-\frac{9}{4}x+30-\frac{1}{4}x=70$$

$$-\frac{5}{2}x=-230$$

$\therefore x=92$ (bottles of water)

$\therefore$ there are 28 cups of coffee

11.3.1	$7.6! = 5\,040$ **OR/*OF*** $7! = 5\,040$
11.3.2	Possible outcomes: $A \times 6 \times 5 \times B \times 4 \times 3 \times 2 \times 1$ $A \times 6 \times 5 \times 4 \times B \times 3 \times 2 \times 1$ $A \times 6 \times 5 \times 4 \times 3 \times B \times 2 \times 1$ $A \times 6 \times 5 \times 4 \times 3 \times 2 \times B \times 1$ $A \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times B$ $5 \times 6! + 4 \times 6! + 3 \times 6! + 2 \times 6! + 1 \times 6!$ $= 6!(5+4+3+2+1)$ $= 6!(15)$ P(two or more runners finishing after Andrew) $= \frac{6!(15)}{8!}$ $= \frac{15}{56}$ $= 0,27$ **OR/*OF*** $\frac{8! - (7!.2 + 2.6.6!)}{8!.2}$ $= \frac{15}{56}$

Paper B

10.1

$P(A \text{ or } B) = P(A) + P(B)$

$P(B) = P(A \text{ or } B) - P(A)$

$= 0,79 - 0,42$

$= 0,37$

10.2

People pay: R2 600

$0,7 \times R2\ 600 = R1\ 820$

Total value of pay-outs = R2 600 – R1 820

= R780

$$P(\text{someone to win}) = \frac{16}{52} \times \frac{1}{2}$$

$$= \frac{2}{13}$$

$$\text{Total number of people winning} = \frac{2}{13} \times 260$$

$$= 40$$

$$\therefore \text{Pay-out} = \frac{R780}{40}$$

= R19,50 per person winning

OR/*OF*

People pay: R2 600

Total value of pay-outs

$= 0,3 \times R2\ 600$

= R780

$$P(\text{someone to win}) = \frac{16}{52} \times \frac{1}{2}$$

$$= \frac{2}{13}$$

$$\text{Total number of people winning} = \frac{2}{13} \times 260$$

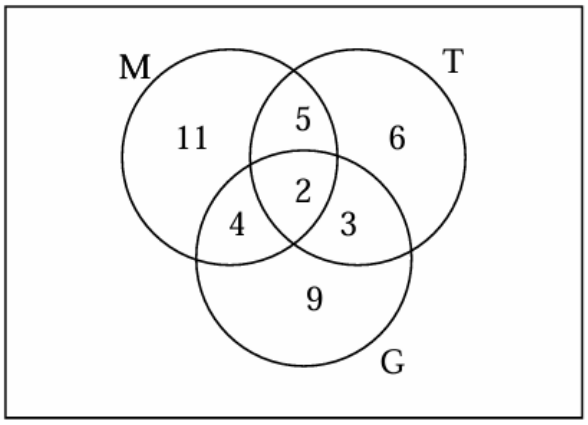
$$= 40$$

$$\therefore \text{Pay-out} = \frac{R780}{40}$$

= R19,50 per person winning

11.1	$\underline{1} \times \underline{9} \times \underline{9} = 81$ (5) 500 cannot be included $\therefore$ 80 possibilities $\underline{4} \times \underline{1} \times \underline{9} = 36$ (5) $\underline{4} \times \underline{9} \times \underline{1} = 36$ (5) Total possibilities $= 80 + 36 + 36$ $= 152$ **OR** 501 to 599: $99 - 10 - 9 = 80$ possibilities 601 to 699: $9 + 9 = 18$ possibilities of having a 5 in the 10's digit or units digit. Thus 601 to 699 $= 4 \times 18$ Total possibilities $= 4 \times 18 + 80$ $= 152$
11.2	P(not having such a number) $= 1 - \frac{152}{499}$ $= \frac{347}{499}$ $= 0,70$

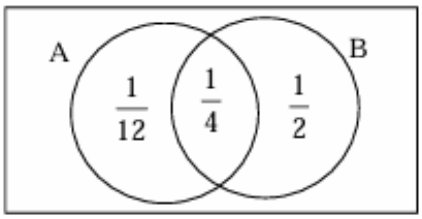
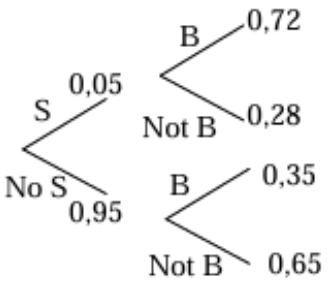
Paper C

11.1	
11.2	$P(\text{at least two subjects}) = \frac{4+2+5+3}{40} = \frac{14}{40} = \frac{7}{20} = 0,35$
11.3	$P(M) \times P(T) = \frac{22}{40} \times \frac{16}{40} = \frac{11}{50} = 0,22$ $P(M \text{ and } T) = \frac{7}{40} = 0,175$ $P(M) \times P(T) \neq P(M \text{ and } T)$ Events M and T are not independent
12.1	$26 \times 10 \times 26 \times 10 = 67\ 600$
12.2	$18 \times 9 \times 19 \times 5 = 15\ 390$
12.3	$24 \times 9 \times 25 \times 5 = 27\ 000$ $\frac{27\ 000 - 15\ 390}{15\ 390} \times 100 = 75,44\%$

Paper D

11.1	$P(A) + P(B) = 0,52$ $0,4 + P(B) = 0,52$ $P(B) = 0,12$
11.2.1	$P(\text{sandwich}) = \frac{4}{25}$ **OR/*OF*** $0,02 + 0,01 + 0,04 + 0,09 = \frac{4}{25} = 0,16$
11.2.2	$P(\text{at least two events}) = 0,02 + 0,01 + 0,03 + 0,04$ $= 0,1$
11.2.3	$P(\text{not any}) = 1 - (0,1 + 0,04 + 0,09 + 0,2)$ $= 0,57$
11.3.1	$7! = 5040$
11.3.2	$P(\text{4 players alphabetically}) = \frac{1}{7 \times 6 \times 5 \times 4} = \frac{1}{840}$
11.3.3	<table><tr><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td></tr></table> F arrangements: 4! M arrangements: 5 options with 3 males = $5 \times 4 \times 3$ $4! \times 5 \times 4 \times 3$ $= 1\ 440$ **OR/*OF*** 10 Options: F **M** F **M** F **M** F **M** F **M** F **M** F F F F **M** F **M** F **M** F **M** F **M** F F **M** **M** F **M** F F **M** F **M** F **M** F F F **M** F **M** F F **M** F **M** **M** F F **M** F F **M** **M** F F **M** F **M** F **M** F F F **M** F **M** Hence $10 \times 4! \times 3! = 1440$

Paper E

10.1.1	$P(A \text{ and } B) = P(A) \times P(B)$ $= \frac{1}{3} \times \frac{3}{4}$ $= \frac{1}{4}$
10.1.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ $= \frac{5}{6}$ **OR/*OF***  $P(A \text{ or } B) = \frac{1}{12} + \frac{1}{4} + \frac{1}{2} = \frac{5}{6}$
10.2.1	
10.2.2	P(NOT below 0^o) = P(S; NOT below 0^o) + P(NS ; NOT below 0^o) = 0,05 × 0,28 + 0,95 × 0,65 = 0,6315
10.3.1	n(S) = 10!

10.3.2

4 Options;

$\mathbf{2} \times 8 \times 7 \times 6 \times 5 \times 4 \times \mathbf{1} \times 3 \times 2 \times 1 = 80\,640$

$8 \times \mathbf{2} \times 7 \times 6 \times 5 \times 4 \times 3 \times \mathbf{1} \times 2 \times 1 = 80\,640$

$8 \times 7 \times \mathbf{2} \times 6 \times 5 \times 4 \times 3 \times 1 \times \mathbf{1} \times 1 = 80\,640$

$8 \times 7 \times 6 \times \mathbf{2} \times 5 \times 4 \times 3 \times 2 \times 1 \times \mathbf{1} = 80\,640$

Total number of possibilities = 322 560

$$P(\text{5 learners in between}) = \frac{322\,560}{10!} = \frac{4}{45}$$

OR/*OF*

$\mathbf{2} \times 8 \times 7 \times 6 \times 5 \times 4 \times \mathbf{1} \times 3 \times 2 \times 1$

4 possible starting positions

$\therefore 4(2 \times 8! \times 1) = 322\,560$

$8(8!) = 322\,560$

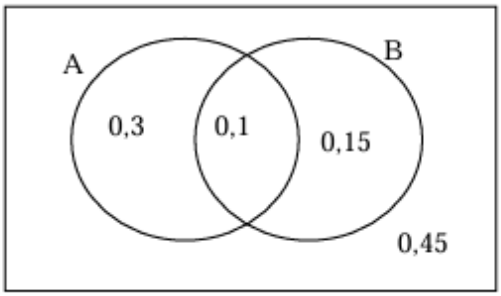
$$P(\text{5 learners in between}) = \frac{322\,560}{10!} = \frac{4}{45}$$

Paper F

10.1.1(a)	$y = 1 - 0{,}893 = 0{,}107$ (0,11)
10.1.1(b)	$x = 0{,}893 - 0{,}733$ $= 0{,}16$
10.1.2	P(at least 2 events) = 0,1 + 0,15 + 0,16 + 0,2 = 0,61
10.1.3	P(B) = 0,643 P(C) = 0,56 P(B and C) = 0,36 P(B) × P(C) = 0,643 × 0,56 = 0,36 ∴ P(B and C) = P(B) × P(C) ∴ B and C are independent
10.2.1	$7 \times 6 \times 5 \times 4 = 840$
10.2.2	start with 5, 7, 9 or start with 6 or start with 8 $(3 \times 5 \times 1 \times 3) + (1 \times 5 \times 1 \times 2) + (1 \times 5 \times 1 \times 2)$ $= 45 + 10 + 10$ $= 65$ $P = \frac{65}{840} = \frac{13}{168} = 0{,}08$ **OR/*OF*** ends in 4 or ends in 6 or ends in 8 $(5 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1)$ $= 25 + 20 + 20$ $= 65$ $P = \frac{65}{840} = \frac{13}{168} = 0{,}08$

Paper G

10.1	P(same day) $= \frac{4}{16}$ or $\frac{1}{4}$ or 0,25 or 25%
10.2	P(2 consecutive days) $= \frac{3 \times 2}{16} = \frac{3}{8}$

11.1.1	$P(A) \times P(B)$ independent events $= 0,40 \times 0,25 = 0,1$ 
11.1.2	$P(\text{A or not B}) = P(\text{A}) + P(\text{not B}) - P(\text{A and not B})$ $= 0,4 + 0,75 - 0,3$ $= 0,85$ **OR/OF** $P(\text{A or not B}) = 1 - P(\text{only B})$ $= 1 - 0,15$ $= 0,85$ **OR/OF** From Venn diagram: 0,3 + 0,1 + 0,45 = 0,85
11.2	$(5 \times 1 \times 5) + (5 \times 1 \times 6) + (5 \times 1 \times 6) + (5 \times 1 \times 5) = 110$ $110 \times 5 = 550 > 500$ Not possible, because not enough space **OR/OF** $(5 \times 2 \times 5) + (5 \times 2 \times 6) = 110$ $110 \times 5 = 550 > 500$ Not possible because not enough space

OR

$5 \times 4 \times 6 = 120$

$5 \times 2 = 10$

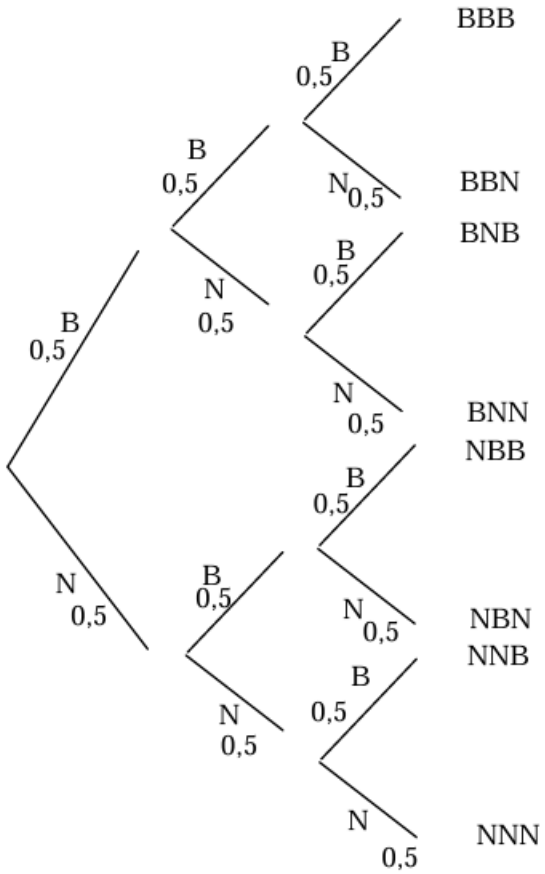
$\therefore 120 - 10 = 110$

$110 \times 5 = 550 > 500$

Not possible because not enough space

Paper H

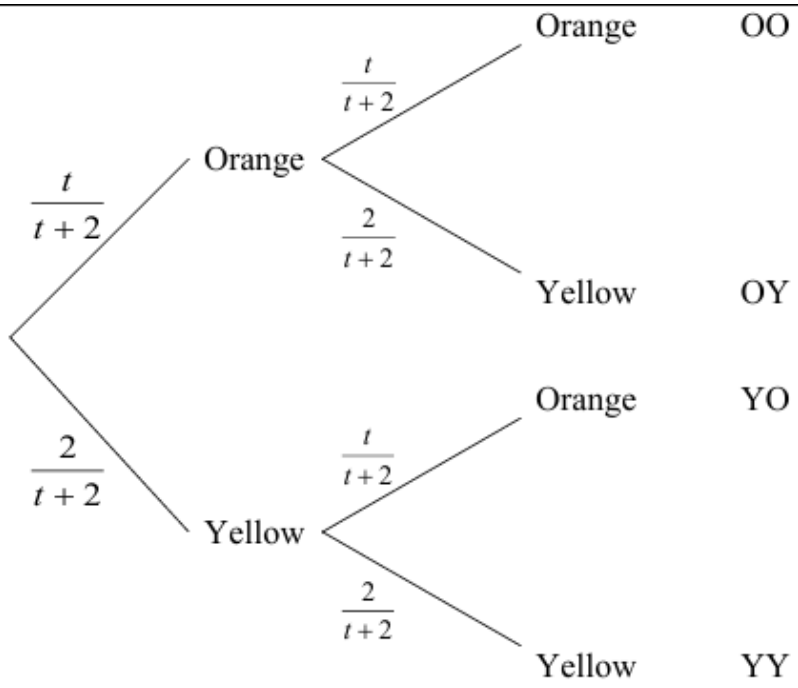
10.1	10^{10} or 10 000 000 000
10.2.1	$\underline{8} \times \underline{10} \times \underline{10}$ (Area) $\times$ $\underline{8} \times \underline{8} \times \underline{10}$ (exchange) $\times$ $\underline{2} \times \underline{10} \times \underline{10} \times \underline{10}$ (number) No. of valid 10-digit numbers $= (8 \times 10 \times 10) \times (8 \times 8 \times 10) \times (2 \times 10 \times 10 \times 10)$ $= 1{,}024 \times 10^9$
10.2.2	Probability $= \dfrac{1{,}024 \times 10^9}{10^{10}}$ $= \dfrac{64}{625} = 0{,}1024 = 10{,}24\%$

11	First shot / Second shot / Third shot 
11.1	P (Bull's eye first shot and second shot) $= 0,5 \times 0,5$ $= 0,25$ or $\frac{1}{4}$
11.2	P (Bull's eye at least twice in 3 shots) $= (0,5 \times 0,5 \times 0,5) + (0,5 \times 0,5 \times 0,5) + (0,5 \times 0,5 \times 0,5) + (0,5 \times 0,5 \times 0,5)$ $= 0,125 + 0,125 + 0,125 + 0,125$ $= 0,5$ or $\frac{1}{2}$
11.3	Person shoots first: $(0,5) + (0,5)^3 + (0,5)^5 + \ldots$ $P = \frac{a}{1-r}$ $P = \frac{0,5}{1-0,25}$ $P = \frac{2}{3} = 0,67$

Paper I

11.1	$P(A) \times P(B)$ $= 0,2 \times 0,63$ $= 0,126$ i.e. $P(A) \times P(B) = P(A \text{ and } B)$ Therefore A and B are independent/*Dus is A en B onafhanklik*
11.2.1	$7^7 = 823\ 543$
11.2.2	7!=5040
11.2.3	There are 3 vowels $\Rightarrow$ 3 options for first position There are 4 consonants $\Rightarrow$ 4 options for last position The remaining 5 letters can be arranged in $5 \times 4 \times 3 \times 2 \times 1$ ways $3 \times (5 \times 4 \times 3 \times 2 \times 1) \times 4 = 1440$ *Daar is 3 klinkers* $\Rightarrow$ *3 opsies vir die eerste posisie* *Daar is 4 konsonante* $\Rightarrow$ *4 opsies vir die laaste posisie* *Die oorblywende 5 letters kan as volg gerangskik word* $5 \times 4 \times 3 \times 2 \times 1$ ways/*maniere* $3 \times (5 \times 4 \times 3 \times 2 \times 1) \times 4 = 1440$

11.3



$$P(\text{Orange},\text{Orange}) + P(\text{Yellow},\text{Yellow}) = \frac{52}{100}$$

$$\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right) + \left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right) = \frac{52}{100}$$

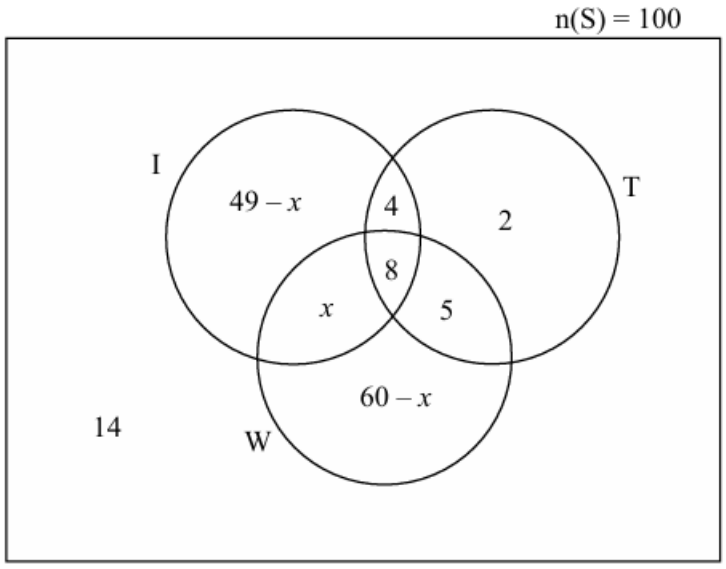
$$\frac{t^2}{t^2+4t+4} + \frac{4}{t^2+4t+4} = \frac{13}{25}$$

$$25\left(t^2+4\right) = 13\left(t^2+4t+4\right)$$
$$3t^2 - 13t + 12 = 0$$
$$(3t-4)(t-3) = 0$$
$$t = 3$$

There are 3 orange balls in the bag/*Daar is 3 oranje balle in die sak*

Paper J

10.1



10.2 $(49-x)+x+8+4+5+2+(60-x)+14=100$

$-x+142=100$

$x=42$

10.3 P (use only one application) $=\frac{7+2+18}{100}$

$=\frac{27}{100}$ or 27%

11.1 5 x 5 x 10 x 9
= 2250

11.2

No of digits used	Letters	Digits	Total
1	5 x 5	10	250
2	5 x 5	10 x 9	2 250
3	5 x 5	10 x 9 x 8	18 000
4	5 x 5	10 x 9 x 8 x7	126 000
5	5 x 5	10 x 9 x 8 x7 x 6	756 000

Codes of two letters and five digits will ensure unique numbers for 700 000 clients.

SECTION B: PAPER 2

STATISTICS

Paper A

1.1.1	$\bar{y} = \frac{155}{10}$ $= 15,5$
1.1.2	SD = 4,59
1.2	$\bar{y} - \text{SD}$ $= 15,5 - 4,59$ $= 10,91$ $\therefore$ 10 – 2 = 8 learners
1.3	$a = 1,7709\ldots$ $b = 0,2243\ldots$ $\hat{y} = 1,77 + 0,22x$
1.4	$\hat{y} = 1,77 + 0,22(72)$ $= 17,61$ ≈ 18 votes **OR/*OF*** $\hat{y} = 17,92 \approx 18$ votes
1.5.1	Points are all scattered therefore low correlation and unrealistic prediction./*Punte is versprei daarom 'n lae korrelasie en onrealistiese voorspelling.*
1.5.2	$r = 0,98$/correlation very strong/*korrelasie baie sterk* $\therefore$ a reliable prediction/*'n betroubare voorspelling*

2.1	60 employees
2.2	$20 < x \leq 25$
2.3	$60 - 34$ $= 26$ employees
2.4	$\text{Salary} = \frac{100}{7} \times 2400$ $\text{Salary} = \text{R}34\ 285{,}71$
2.5	$\therefore$ Ogive/Cumulative frequency graph will shift to the right/will become steeper. $\therefore$ *Ogief/Kumulatiewe frekwensie grafiek sal na regs skuif/sal steiler wees.*

Paper B

1.1	$a = -23{,}846\ldots$ $b = 0{,}227\ldots$ $\hat{y} = -23{,}85 + 0{,}23x$
1.2	$\hat{y} = -23{,}85 + 0{,}23(550)$ $y = 102{,}65$ **OR** $y = 101{,}02$
1.3	$r = 0{,}98$
1.4	Very strong positive correlation

50	100	130	150	180	190	200	200

1.5.1	$\overline{x} = \frac{1200}{8}$ $\overline{x} = 150$ **OR** $\overline{x} = 150$
1.5.2	$\sigma = 50,50$
1.5.3	$\overline{x} - \sigma$ $= 150 - 50,50$ $= 99,50$ $\therefore$ 1 stop

2.1

Number of glasses of water per day	Number of staff members	Cumulative frequency
$0 \leq x < 2$	5	5
$2 \leq x < 4$	15	20
$4 \leq x < 6$	13	33
$6 \leq x < 8$	5	38
$8 \leq x < 10$	2	40

2.2	40 staff members
2.3	33 staff members

2.4 $$\bar{x} = \frac{\left(1\times\left(5+\frac{k}{2}\right)\right)+(3\times15)+\left(5\times\left(13+\frac{k}{2}\right)\right)+(7\times5)+(9\times2)}{40+k} = 4$$

$$5+\frac{k}{2}+45+65+\frac{5k}{2}+35+18 = 160+4k$$

$$3k+168 = 160+4k$$

$$k = 8$$

OR

$$\bar{x} = \frac{(1\times5)+(15\times3)+(13\times5)+(5\times7)+(2\times9)}{40}$$

$$= 4{,}2$$

$$\bar{x}_{\text{old}} - \bar{x}_{\text{current}} = 4{,}2-4$$

$$= 0{,}2$$

$$\therefore 0{,}2\times40$$

$$= 8 \text{ teachers}$$

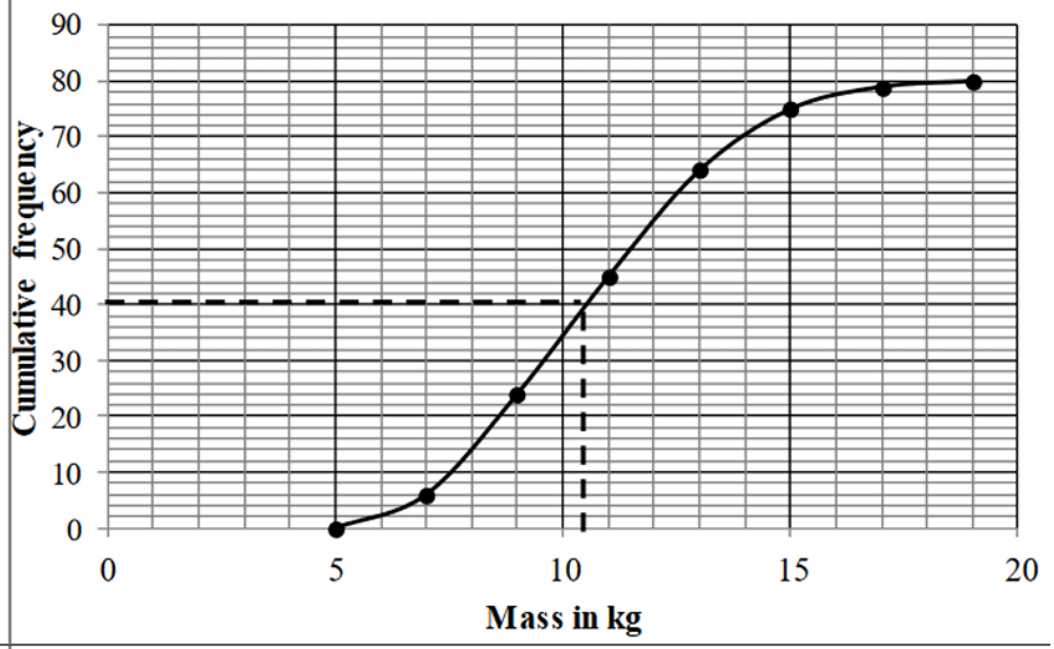
Paper C

1.1 Modal class: $9 < m \leq 11$

1.2

Mass (in kg)	Frequency	Cumulative frequency
$5 < m \leq 7$	6	6
$7 < m \leq 9$	18	24
$9 < m \leq 11$	21	45
$11 < m \leq 13$	19	64
$13 < m \leq 15$	11	75
$15 < m \leq 17$	4	79
$17 < m \leq 19$	1	80

1.3



1.4 Median mass: 10,5 kg

1.5.1

$$\bar{x} = \frac{(6\times 6+18\times 8+21\times 10+19\times 12+11\times 14+4\times 16+1\times 18)}{80}$$

$$= \frac{854}{80}$$

$$= 10{,}68$$

1.5.2

Learners' bags are heavier than the stipulated international guideline.

Estimated mean = 10,68 kg

10% of 80 kg

= 8 kg

10,68 kg > 8 kg

OR/ *OF*

Learners' bags are heavier than the stipulated international guideline.

$$\text{Estimated mean} = \frac{10{,}68}{80}\times 100$$

$$= 13{,}35\%$$

13,35% > 10%

2.1	$a = 634{,}382\ldots$ $b = 32\ 189{,}263\ldots$ $\hat{y} = 634{,}38 + 32189{,}26x$
2.2	$\hat{y} = 634{,}38 + 32189{,}26(0{,}25)$ $= \text{R}8\ 681{,}70$ **OR/*OF*** $\hat{y} = \text{R}8\ 681{,}70$ (if using calculator)
2.3	Average price increase $= \text{R}\dfrac{32189{,}26}{20}$ per 0,05 carat $= \text{R}1\ 609{,}46$ per 0,05 carat **OR/*OF*** Average price increase $= 0{,}05 \times 32\,189{,}26$ $= \text{R}1\ 609{,}46$ per 0,05 carat **OR/*OF*** at 0,3: $\hat{y} = \text{R}10\,291{,}16$ $\therefore$ Average price increase $= 10\ 291{,}16 - 8\ 681{,}70$ $=$ R1 609 46 per 0,05 carat
2.4	The point (0,35 ; 11500) is closer to the least squares regression line.

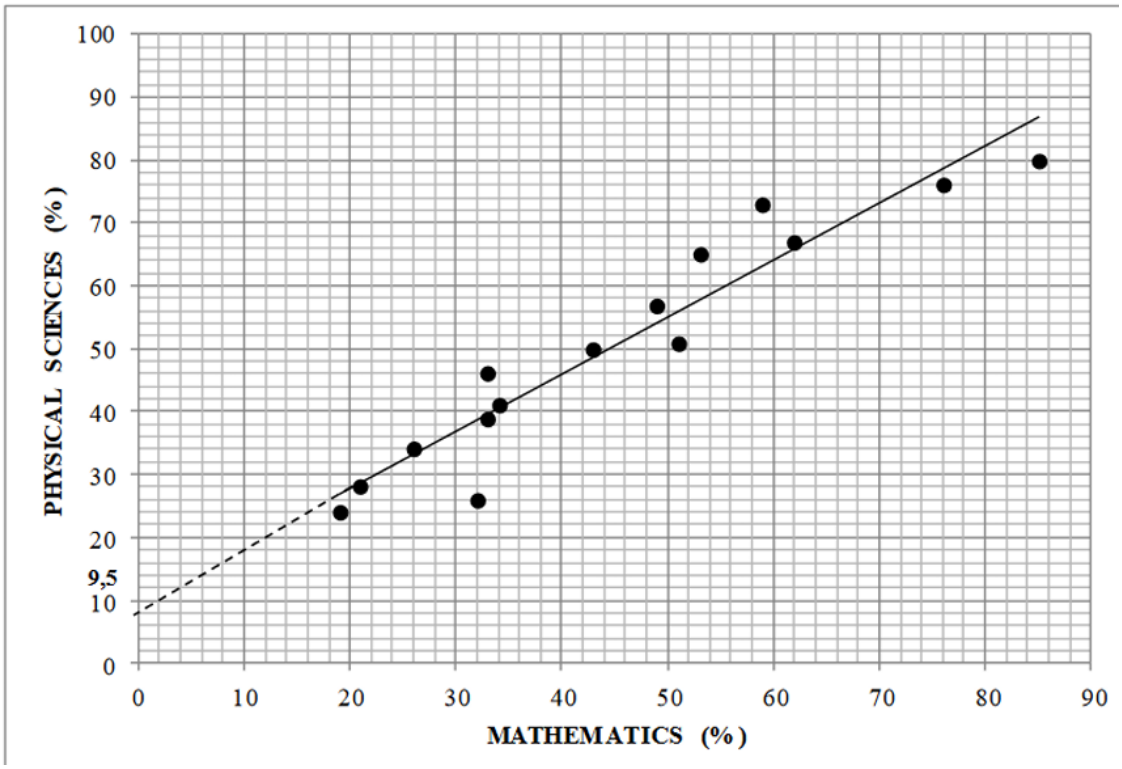
Paper D

1.1 $a = 9{,}5$

$b = 0{,}909.. = 0{,}91$

$\hat{y} = 9{,}5 + 0{,}91x$

1.2



1.3 Final exam mark $\approx$ 72,22% (calculator)

OR

$\hat{y} = 9{,}5 + 0{,}91(69)$

$\approx$ 72,29%

1.4 $r = 0{,}95$

1.5 There is a **very strong positive** correlation between the Mathematics and Physical Sciences mark.

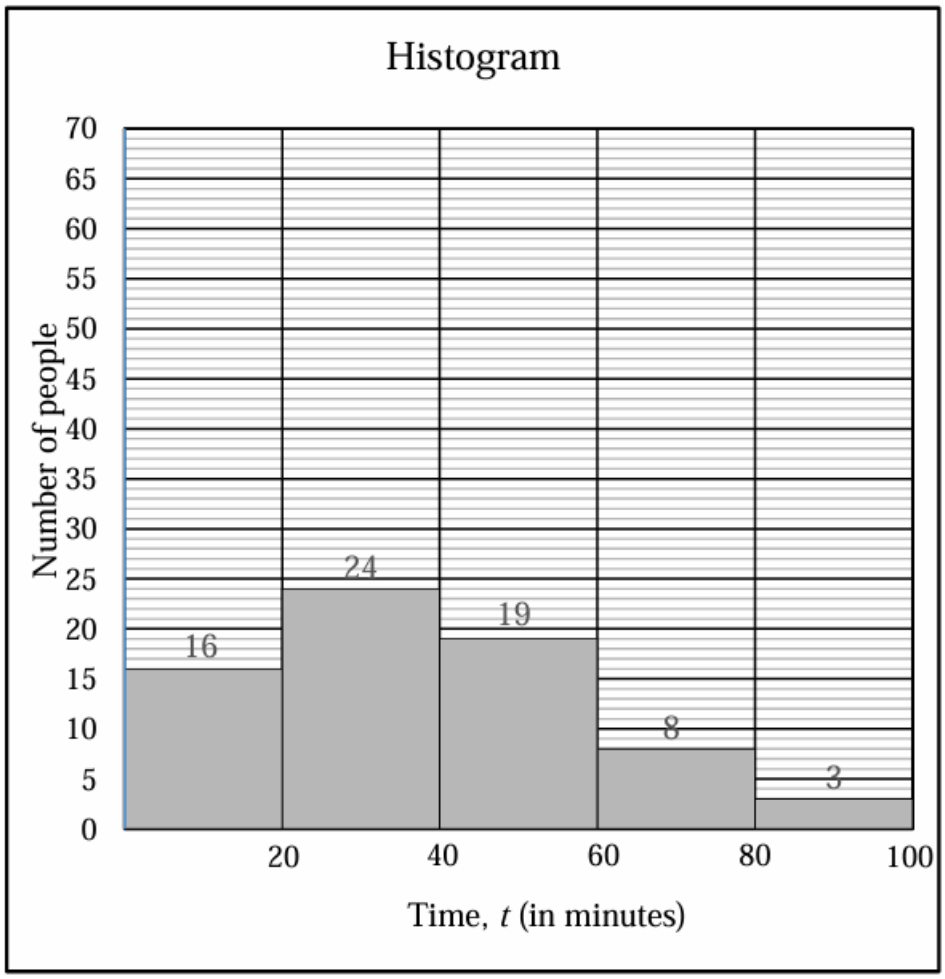
1.6 The teacher concludes that the higher the learners' Mathematics marks, the higher the learners' Physical Sciences marks.

2 018	**2 175**	**2 182**	**2 215**	**2 254**	**2 263**	**2 267**	**2 271**	**2 293**	**2 323**	**2 334**	**2 346**

2.1	July / *Julie*
2.2	$\bar{x} = \frac{26941}{12}$ $= 2\,245{,}083..\ \approx 2\,245{,}08$ aircraft landings
2.3	Standard deviation for landings at the King Shaka International airport: $\sigma = 86{,}30$
2.4	$(\bar{x} - \sigma\,;\bar{x} + \sigma) = (2\,245{,}08 - 86{,}30\ ;\ 2\,245{,}08 + 86{,}30)$ limit = (2 158,78 ; 2 331,38) There were **9** months when the aircraft arrivals at the King Shaka International airport were within one standard deviation of the mean.
2.5	The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of arrivals at the King Shaka International Airport **OR C.**

Paper E

1.1	$a = 331\,397{,}20$ $b = -22\,988{,}32$ $\hat{y} = 331\,397{,}20 - 22\,988{,}32x$
1.2	$\hat{y} = 331\,397{,}20 - 22\,988{,}32(5)$ $= 216\,455{,}60$ **OR/*OF*** $\hat{y} = 216\,455{,}61$ (calculator)
1.3	The strong correlation $(r = -0{,}95)$ suggests that the data points lie close to the regression line. Therefore, the prediction will be valid./ *'n Sterk korrelasie* $(r = -0{,}95)$ *dui aan dat die punte naby aan die regressielyn lê.* *Dus, die voorspelling is geldig.*
1.4	The average decrease per year is R22 988,32. *Die gemiddelde afname per jaar is* R 22 988,32.

2.1.1	70
2.1.2	No. of people = 67 – 40 = 27
2.1.3	
2.1.4	Skewed to the right **OR** positively skewed *Skeef na regs* **OF** *positief skeef*

2.2

11	14	19	20	8	10	2	14

$$\frac{11+14+19+20+8+10+2+14+x}{9} = 12$$

$$x+98 = 108$$

$$x = 10$$

The 9th player scored 10 points

$$\sigma = 5,23 \qquad (5, 22812)$$

$$(\bar{x}-\sigma\ ;\ \bar{x}+\sigma) = (12-5,23;\ 12+5,23)$$

$$= (6,77;17,23)$$

3 players' points were outside one standard deviation of the mean.
3 *spelers se punte aangeteken lê buite een standaardafwyking van die gemiddeld.*

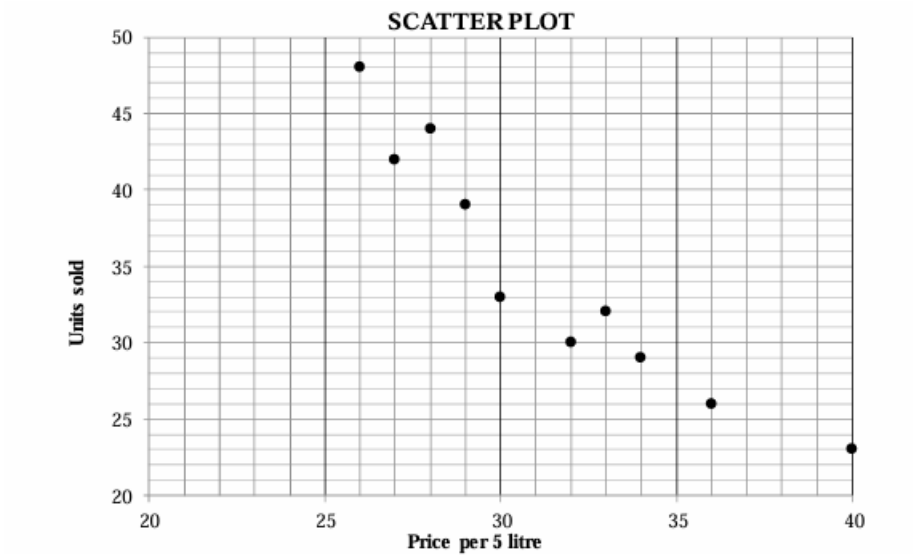
Paper F

1.1	$\bar{x} = \frac{1\,581}{31}$ $= 51$ **OR / *OF*** $\bar{x} = 51$ (calculator method / *sakrekenaar metode*)
1.2	$\therefore$ skewed to the left
1.3	Physical Sciences performed better. Q_1 is 40% in Physical Sciences and 28% in Mathematics which indicates the lower 25% of the class performed much better in Physical Sciences than in Mathematics. *Fisiese Wetenskappe presteer beter.* *Q_1 is 40% in Fisiese Wetenskappe en 28% in Wiskunde wat aandui dat die onderste 25% van die klas heelwat beter presteer in Fisiese Wetenskappe as in Wiskunde.*
1.4	Accept any mark between 40 – 50. *Aanvaar enige punt tussen 40 – 50 .*
1.5	The greatest difference is 87%−71%=16%

2.1	$a = 12,41$ $b = 0,49$ $\hat{y} = 12,41 + 0,49x$
2.2	$\hat{y} = 12,41 + 0,49x$ $= 12,41 + 0,49(150)$ $= 85,91 \approx 86\%$ **OR/OF** $\hat{y} = 85,17$
2.3	$\hat{y} = 12,41 + 0,49x$ The y-intercept is 12,41 which means that a learner who did not begin the exam achieved 12,41%. This is clearly impossible.
2.4	10,28
2.5	$63,9 - \sigma = p$ $63,9 + \sigma = 103,59$ $127,92 = p + 103,59$ $p = 24,33$ **OR / *OF*** $\sigma = 103,59 - 63,96$ $= 39,63$ $p = 63,96 - 39,63$ $= 24,33$

Paper G

1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$
1.1.2	$\sigma = 10{,}1707 \approx 10{,}17$
1.1.3	$\bar{x} + \sigma = 32{,}17$ $\therefore$ 5 days
1.2	$22 \times 18 = 396$ ordered/*bestel* $20 \times 18 = 360$ sold/*verkoop* Total not sold/*Totaal nie verkoop nie*: 36 **OR/*OF*** 22–20 = 2 $2 \times 18 = 36$
1.3.1	Option B/*Opsie B* Any one of the following reasons/*Enige een van die vlg redes*: • Median/*Mediaan* = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right
1.3.2	Data is positively skewed/skewed to the right *Data is positief skeef/skeef na regs*

2.1	
2.2	$a = 90{,}478... \approx 90{,}48$ $b = -1{,}773... \approx -1{,}77$ $\hat{y} = 90{,}48 - 1{,}77x$
2.3	$y = 23{,}069\ldots \approx 23{,}07$ units/*eenhede* (calculator/*sakrekenaar*) **OR/*OF*** $y = 90{,}48 - 1{,}77(38)$ $y = 23{,}22$ units/*eenhede*
2.4	$r = -0{,}94$ The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold $\therefore$ there is a good chance of the prediction being accurate./ *Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan* $\therefore$ *daar is 'n goeie kans dat die voorspelling akkuraat is*

Paper H

1.1 45 children

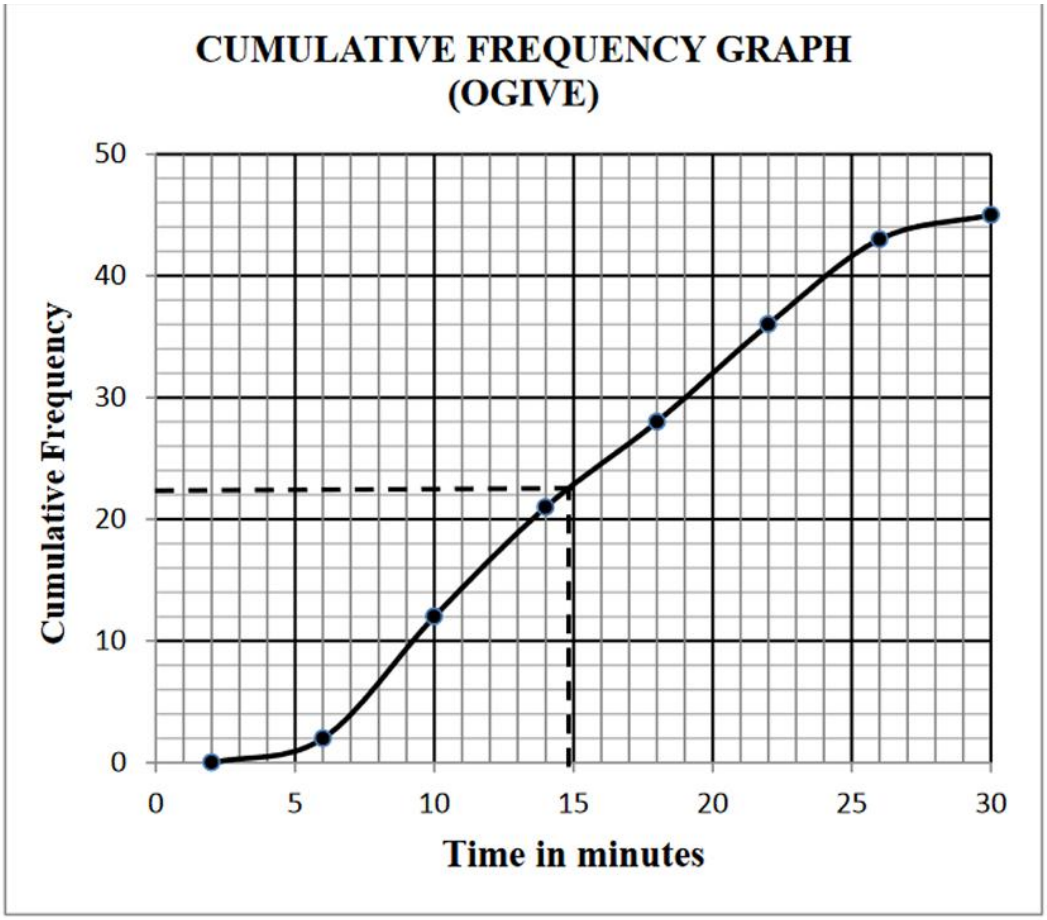
1.2 $\bar{x} = \frac{\sum fx}{n} = \frac{(4\times 2)+(8\times 10)+(12\times 9)+(16\times 7)+(20\times 8)+(24\times 7)+(28\times 2)}{45}$

$\bar{x} = \frac{692}{45}$ OR $\bar{x} = 15{,}38$ minutes

1.3

Time taken (t) (in minutes)	Number of children	Cumulative frequency
$2 < t \leq 6$	2	2
$6 < t \leq 10$	10	12
$10 < t \leq 14$	9	21
$14 < t \leq 18$	7	28
$18 < t \leq 22$	8	36
$22 < t \leq 26$	7	43
$26 < t \leq 30$	2	45

1.4

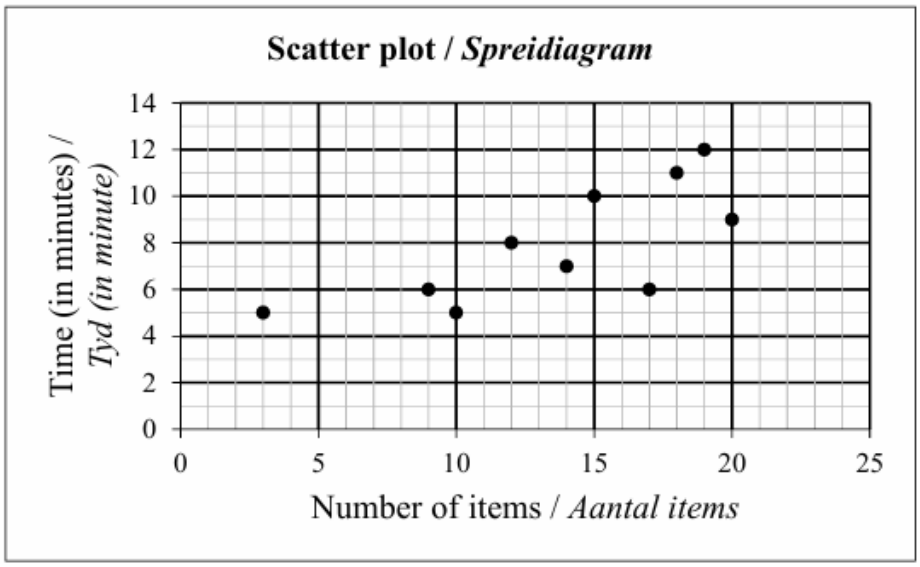


1.5 On graph at the *y*-value of 22,5 or 23
Median = ± 15 minutes.

2.1	$a = 12{,}44$ $b = 0{,}98$ $y = 12{,}44 + 0{,}98x$
2.2.1	$\text{Percentage} = \frac{15}{50} \times 100$ $= 30\%$
2.2.2	$\hat{y} = 12{,}44 + 0{,}98x$ $\hat{y} = 12{,}44 + 0{,}98(30)$ $\hat{y} = 41{,}84$ $= 42$ **OR** $\hat{y} = 41{,}87$ (if using calculator) $\hat{y} = 42$ **OR** $\hat{y} = \frac{21}{50}$
2.3.1	standard deviation =13,88
2.3.2	$x = 50{,}67 - 45{,}67$ $= 5\%$

Paper I

1.1	$\overline{x} = \dfrac{8554}{15}$ $= 570{,}27$
1.2	$\sigma = 291{,}03$
1.3	$(279{,}24\ ;\ 861{,}3)$ $\therefore 10$ premiums
1.4	$\dfrac{\left(1791 \times \dfrac{118}{100}\right) + \left(6763 \times \dfrac{k+100}{100}\right)}{15} = 686{,}44$ $6763 \times \dfrac{k+100}{100} = 8183{,}22$ $\dfrac{k+100}{100} = 1{,}209\ldots$ $k + 100 = 120{,}999\ldots$ $k = 21\%$

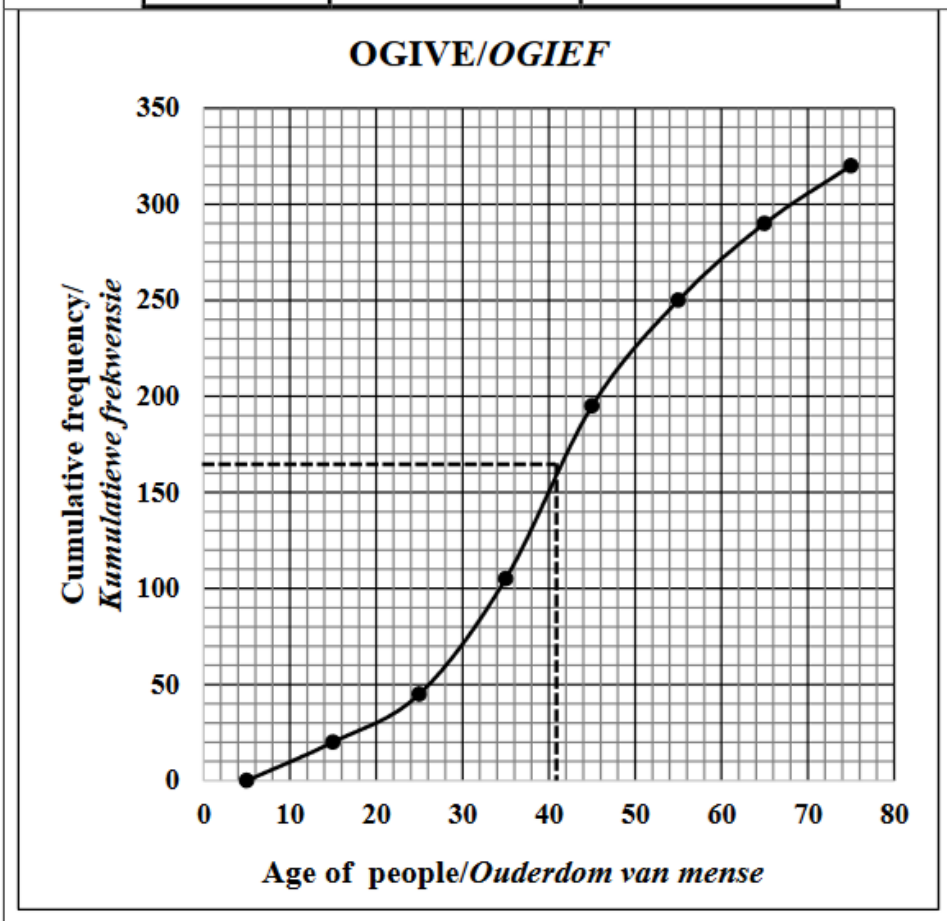
2.1	
2.2	$a = 3{,}079\ldots$ $b = 0{,}351\ldots$ $\hat{y} = 3{,}08 + 0{,}35x$
2.3	$r = 0{,}74$
2.4	$y = 3{,}08 + 0{,}35(13)$ $y = 7{,}63$ **OR** $y = 7{,}65$ (calculator)
2.5	It does not make sense to **pack 0** items in **3,08 minutes./** *Dit maak nie sin dat **0 items** in **3,08 minute** gepak kan word nie.*

Paper J

1.1.1	$a = 1730{,}22$ $b = 13{,}96$ $\hat{y} = 1730{,}22 + 13{,}96x$
1.1.2	$\hat{y} = 1730{,}22 + 13{,}96x$ $\hat{y} = 1730{,}22 + 13{,}96(28\,500)$ $\hat{y} = \text{R}399\,590{,}22$ **OR/*OF*** $\hat{y} = \text{R}399\,599{,}64$ (calc)
1.1.3	$r = 0{,}98002\ldots$ $r = 0{,}98$
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. /
1.2.1	$\bar{x} = \frac{1\,552\,195}{9}$ $\bar{x} = 172\,466{,}11$
1.2.2	$\sigma = 56\,950{,}09$
1.2.3	$\bar{x} + \sigma$ $= 172\,466{,}11 + 56\,950{,}09$ $= 229\,416{,}20$ 2 years/*jaar*
2.1	$35 < x \leq 45$
2.2	320 people/*mense*

2.3

AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY
$5 < x \leq 15$	20	20
$15 < x \leq 25$	25	45
$25 < x \leq 35$	60	105
$35 < x \leq 45$	90	195
$45 < x \leq 55$	55	250
$55 < x \leq 65$	40	290
$65 < x \leq 75$	30	320



2.4 Median = 41

ANALYTICAL GEOMETRY

Paper A

3.1.1	$m_{AB}=\dfrac{2-(-4)}{4-6}$ **OR** $m_{AB}=\dfrac{-4-2}{6-4}$ $m_{AB}=-3$
3.1.2	$\tan\alpha=m_{AB}=-3$ $\alpha=108{,}43°$
3.1.3	$T\left(\dfrac{x_1+x_2}{2};\dfrac{y_1+y_2}{2}\right)$ $T\left(\dfrac{-2+6}{2};\dfrac{-3-4}{2}\right)$ $T\left(2;\dfrac{-7}{2}\right)$
3.1.4	$5(0)-6y=8$ $y=-\dfrac{4}{3}$ $S\left(0;\dfrac{-4}{3}\right)$
3.2	$m_{CD}=m_{AB}=-3$ $-3=-3(-2)+c$ OR $y-(-3)=-3(x-(-2))$ $c=-9$ $y=-3x-9$ $y=-3x-9$
3.3.1	$5x-6y=8$ $y=\dfrac{5}{6}x-\dfrac{8}{6}$ $\tan\theta=m_{AC}=\dfrac{5}{6}$ $\theta=39{,}81°$ $\hat{A}=108{,}43°-39{,}81°$ $=68{,}62°$ $D\hat{C}A=68{,}62°$ [alt ∠s ; DC||AB]

3.3.2 $P(-3;0)$ and $F(1,6\,;\,0)$

Area POSC = Area ΔFPC – Area ΔOFS

$$= \frac{1}{2}(4,6)(3) - \frac{1}{2}(1,6)\left(\frac{4}{3}\right)$$
$$= 6,9 - 1,07$$
$$= 5,83 \text{ units}^2$$

OR/*OF*

$P(-3;0)$

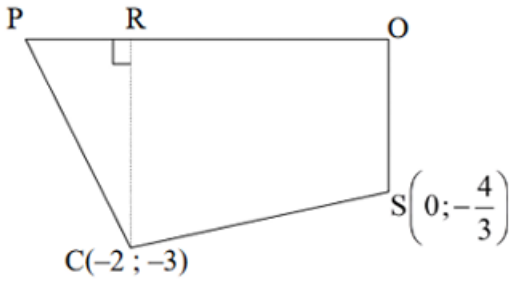
$$FC = \sqrt{\left(-2-\frac{8}{5}\right)^2 + (-3-0)^2} = \frac{3\sqrt{61}}{5}$$

$$\text{Area } \Delta PFC = \frac{1}{2}(PF)(FC)\sin O\hat{F}S$$
$$= \frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39,81°$$
$$= 6,90$$

$$\text{Area } \Delta OFS = \frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$$
$$= 1,07$$

Area POSC $= 6,90 - 1,07$
$= 5,83 \text{ units}^2$

OR

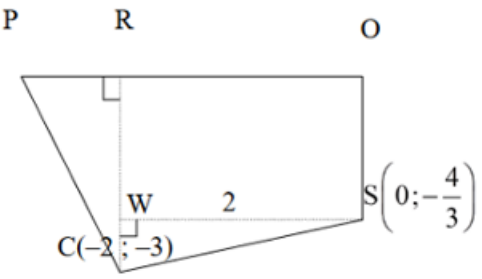


$P(-3;0)$

Area of POSC = Area of OSCR + Area of ΔPRC

$$= \frac{1}{2}\left(\frac{4}{3}+3\right)\times 2 + \frac{1}{2}(1\times 3)$$
$$= \frac{35}{6}$$
$$= 5,83 \text{ units}^2$$

OR



$P(-3;0)$

Area POSC = Area ROSW + Area ΔPRC + Area ΔWSC

$$=\left(\frac{4}{3}\right)(2)+\frac{1}{2}(1)(3)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$$

$$=\frac{35}{6}$$

$$=5,83\text{units}^2$$

OR

$P(-3;0)$

$$\text{Area of } \Delta PSC=\frac{1}{2}(PC)(CS)\sin D\hat{C}A$$

$$=\frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$$

$$=3,833..$$

$$\text{Area of } \Delta POS=\frac{1}{2}(PO)(OS)$$

$$=\frac{1}{2}(3)\left(\frac{4}{3}\right)$$

$$=2$$

Area POSC = 3,833… + 2

$$=5,83\text{units}^2$$

4.1

$P(x;y)$; $N(7;-2)$; $M(3;-5)$

$\frac{x+7}{2}=3 \qquad \frac{y-2}{2}=-5$

$x=-1 \qquad y=-8$

$P(-1;-8)$

4.2.1

$r^2=(7-3)^2+(-2-(-5))^2$ **OR/*OF*** $r^2=(-1-3)^2+(-8-(-5))^2$

$r^2=25$

$(x-3)^2+(y+5)^2=25$

4.2.2

$m_{\text{radius}}=\frac{-5-(-2)}{3-7}=\frac{3}{4}$

$m_{\text{tangent}}=-\frac{4}{3}$ [radius $\perp$ tangent/*raaklyn* $\perp$ *radius*]

$-2=-\frac{4}{3}(7)+c$ **OR** $y-(-2)=-\frac{4}{3}(x-7)$

$c=\frac{22}{3}$ $y=-\frac{4}{3}x+\frac{22}{3}$

$y=-\frac{4}{3}x+\frac{22}{3}$

4.3

$-8=-\frac{4}{3}(-1)+c$

$\therefore\ c=-\frac{28}{3}$

$-\frac{28}{3}<k<\frac{22}{3}$

4.4.1

$AB^2=AM^2-MB^2$

$AB^2=\left[(t-3)^2+(t+5)^2\right]-5^2$

$=t^2-6t+9+t^2+10t+25-25$

$AB=\sqrt{2t^2+4t+9}$

4.4.2

$t=\frac{-4}{2(2)}$

$=-1$

Minimum at $t=-1$

$AB=\sqrt{2(-1)^2+4(-1)+9}$

$AB=\sqrt{7}$

OR

$4t+4=0$

$t=-1$

Minimum at $t=-1$

$AB=\sqrt{2(-1)^2+4(-1)+9}$

$AB=\sqrt{7}$

OR/*OF*

Length of $AB=\sqrt{2t^2+4t+9}$

$$=\sqrt{2\left(t^2+2t+\frac{9}{2}\right)}$$

$$=\sqrt{2\left[(t+1)^2+\frac{7}{2}\right]}$$

$$=\sqrt{2(t+1)^2+7}$$

Minimum at $t=-1$

$AB=\sqrt{2(-1)^2+4(-1)+9}$

$AB=\sqrt{7}$

Paper B

3.1.1

$y=-x-11$

$A(-1\ ;\ t)$

$t=-(-1)-11$

$t=-10$

3.1.2

$\tan\alpha=-1$

$ref.\angle=45°$

$\therefore\alpha=135°$

3.1.3

$\tan 63{,}43°=m_{AC}$

$m_{AC}=2$

3.2

$m_{AC}=2$

$A(-1\ ;-10)$

$y=2x+k$

$-10=2(-1)+k$

$k=-8$

$y=2x-8$

OR/*OF*

$y-y_1=2(x-x_1)$

$y-(-10)=2(x-(-1))$

$y=2x-8$

3.3.1

$y = 2x - 8$

$0 = 2x - 8$

$x_B = 4$

$\frac{x_C + (-1)}{2} = 4$ $\qquad$ $\frac{y_C + (-10)}{2} = 0$

$x_C = 9$ $\qquad$ $y_C = 10$

OR/*OF* by translation / *met translasie*

$A \rightarrow B\ (x; y) \rightarrow (x + 5; y + 10)$

$B \rightarrow C\ (4; 0) \rightarrow (4 + 5;\ 0 + 10) = (9\ ;\ 10)$

3.3.2

$A\hat{B}E = 63,43°$ $\qquad$ [vert. opp ∠'s =]

$\hat{E}_2 = 63,43°$ $\qquad$ [corres. ∠'s, DE || AB]

$\hat{E}_1 = 45°$ $\qquad$ [∠s on a str line]

$F\hat{E}D = 108,43°$

OR/*OF*

$E\hat{A}B = 135° - 63,43°$

$E\hat{A}B = 71,57°$

$D\hat{E}A = E\hat{A}B = 71,57°$

$F\hat{E}D = 108,43°$

OR/*OF*

$A\hat{B}E = 63,43°$ $\qquad$ [vert. opp ∠'s]

$D\hat{E}O = 116,57°$ $\qquad$ [co-int. ∠'s, DE || AB]

$F\hat{E}D = 360° - (116,57° + 135°)$

$= 108,43°$

3.4

$y = 0$

$x_E = -11$

$\frac{x_G + (-11)}{2} = 4$

$x_G = 19$

$(x - 19)^2 + y^2 = 15^2$

$(x - 19)^2 + y^2 = 225$

4.1 $M(-6;-3)$

4.2.1
$$x^2+y^2+24x-10y+153=0$$
$$(x+12)^2+(y-5)^2=-153+144+25$$
$$(x+12)^2+(y-5)^2=16$$
$$r^2=16$$
$$r=4 \text{ units}$$

4.2.2
$$NM=\sqrt{(-12-(-6))^2+(5-(-3))^2}$$

NM = 10 units

SM = 5 units

$\therefore TS = 10-5-4 = 1$ unit

4.3.1 R(–6 ; –8)

$y=-8$

4.3.2
$$m_{NM}=\frac{5-(-3)}{-12-(-6)}$$
$$m_{NM}=-\frac{4}{3}$$
$$m_{tangent}=\frac{3}{4}$$
$$-5=\frac{3}{4}(-17)+c$$
$$c=\frac{31}{4}$$
$$y=\frac{3}{4}x+\frac{31}{4}$$

OR/*OF*

$$y-y_1=\frac{3}{4}(x-x_1)$$
$$y-(-5)=\frac{3}{4}(x-(-17))$$
$$y=\frac{3}{4}x+\frac{31}{4}$$

OR

$$NS = SM = 5$$

$$S\left(\frac{-12-6}{2}\ ;\ \frac{5-3}{2}\right)$$

$$S\ (-9\ ;\ 1)$$

$$m_{SK} = \frac{1-(-5)}{-9+17}$$

$$= \frac{6}{8} = \frac{3}{4}$$

$$y+5 = \frac{3}{4}(x+17)$$

$$y = \frac{3}{4}x + \frac{31}{4} \quad or \quad y = \frac{3}{4}x + 7\frac{3}{4}$$

4.4.1

$$-8 = \frac{3}{4}x + \frac{31}{4}$$

$$-32 = 3x + 31$$

$$3x = -63$$

$$x = -21$$

P(–21 ; –8)

R(–6 ; –8)

PR = PS = 15 units [tangents from same point]

MS = MR = 5 units

Perimeter PSMR = 15 + 15 + 5 + 5

= 40 units

4.4.2

$$\frac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$$

$$\frac{\frac{1}{2}NS.SP}{\frac{1}{2}SP.MS+\frac{1}{2}MR.PR}$$

$$=\frac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$$

$$=\frac{1}{2}$$

OR

$\Delta NPS \equiv \Delta SPM \equiv \Delta MPR$

$$\frac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$$

$$=\frac{1}{2}$$

Paper C

3.1	$SL = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2}$ $SL = \sqrt{(4-(-4))^2 + (5-1)^2}$ $SL = \sqrt{80} = 4\sqrt{5} = 8,94 \text{ units}$
3.2	$m_{SN} = \frac{5-(-3)}{4-(-2)}$ $m_{SN} = \frac{4}{3}$
3.3	$m = \tan\theta = \frac{4}{3}$ $\theta = 53,13°$
3.4	$m_{LN} = \frac{1-(-3)}{-4-(-2)}$ $m_{LN} = -2$ $L\hat{K}O = 116,565...°$ $L\hat{N}S = 116,565...° - 53,13°$ $L\hat{N}S = 63,44°$

OR

$SN = 10 \text{ units}$ $\sin L\hat{N}S = \frac{4\sqrt{5}}{10}$ $L\hat{N}S = 63,44°$

OR

$LN = 2\sqrt{5}$ units

$\tan L\hat{N}S = \dfrac{4\sqrt{5}}{2\sqrt{5}}$

$L\hat{N}S = 63{,}44°$

OR

$SN = 10$ units

$LN = 2\sqrt{5}$ units

$\cos L\hat{N}S = \dfrac{2\sqrt{5}}{10}$

$L\hat{N}S = 63{,}44°$

3.5

$m = \dfrac{4}{3}$

$1 = \dfrac{4}{3}(-4) + c$

$c = \dfrac{19}{3}$

$y = \dfrac{4}{3}x + \dfrac{19}{3}$

OR

$y - 1 = \dfrac{4}{3}(x - (-4))$

$y - 1 = \dfrac{4}{3}x + \dfrac{16}{3}$

$y = \dfrac{4}{3}x + \dfrac{19}{3}$

3.6

$SL = 4\sqrt{5}$

$LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$

$LN = \sqrt{20} = 2\sqrt{5}$

$\text{Area } \Delta LSN = \dfrac{1}{2}\left(4\sqrt{5}\right)\left(2\sqrt{5}\right)$

$= 20 \text{ units}^2$

OR

$SN = 10$ units

$LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$

$LN = \sqrt{20} = 2\sqrt{5}$

$\text{Area } \Delta LSN = \frac{1}{2}(10)\left(2\sqrt{5}\right)\sin 63,44°$

$= 20 \text{ units}^2$

3.7 $\hat{L} = 90°$

SN is a diameter of circle S, L, N [chord subtends 90°

OR converse $\angle$ in semi-circle]

$\text{Centre of circle} = P\left(\frac{4+(-2)}{2}; \frac{5+(-3)}{2}\right)$

$= P(1\,;1)$

OR

Let the coordinates of P be $(a\,;b)$.

Then, PL = PN: $(-4-a)^2 + (1-b)^2 = (-2-a)^2 + (-3-b)^2$

$a - 2b = -1$ .....equation 1

If PS = PN, then: $4a + 2b = 6$..... equation 2

Solving simultaneously yields: $a = 1$ and $b = 1$ and $P(1\,;1)$

OR

If PL = PN, then: $a - 2b = -1$ .....equation 1

If PS = PL, then: $2a + b = 3$.....equation 2

Solving simultaneously yields: $a = 1$ and $b = 1$ and $P(1\,;1)$

3.8 $\hat{LPN} = \theta = 53{,}13°$ [alt ∠s; LP ∥ x –axis]

$\therefore \hat{LPS} = 126{,}87°$

OR

$\hat{LNS} = 63{,}44°$

$\therefore \hat{LPS} = 126{,}88°$ [∠ at centre = 2 ×∠ at circumference]

OR

$\hat{LSN} = 26{,}56°$ [sum of ∠s in Δ]

$\hat{SLP} = 26{,}56°$ [∠s opp equal radii]

$\therefore \hat{LPS} = 126{,}88°$ [sum of ∠s in Δ]

OR

$$\left(4\sqrt{5}\right)^2 = 5^2 + 5^2 - 2(5)(5)\cos\hat{LPS}$$

$$\cos\hat{LPS} = -\frac{3}{5}$$

$\therefore \hat{LPS} = 126{,}87°$

4.1 $D(p\,;\,-2)$

$$x^2 + y^2 = 20$$

$$p^2 + (-2)^2 = 20$$

$$p^2 = 16$$

$$p = \pm 4$$

$$p = 4$$

4.2 $\dfrac{4 + x_F}{2} = 6$ $\qquad$ $\dfrac{-2 + y_F}{2} = 2$

$x_F = 8$ $\qquad$ $y_F = 6$

$F(8;6)$

OR

$x_E - x_D = 6 - 4$ $\qquad$ $y_E - y_D = 2 - (-2)$

$= 2$ $\qquad$ $= 4$

$x_F = 6 + 2 = 8$ $\qquad$ $y_F = 2 + 4 = 6$

$F(8;6)$

4.3

$m_{DE} = \dfrac{-2-2}{4-6}$

$m_{DE} = 2$

$-2 = 2(4) + c$ **OR** $y-(-2) = 2(x-4)$

$c = -10$ $y+2 = 2x-8$

$y = 2x-10$ $y = 2x-10$

OR

$m_{OD} = -\dfrac{2}{4} = -\dfrac{1}{2}$

$\therefore m_{DE} = 2$ [tan $\perp$ radius]

$-2 = 2(4) + c$ **OR** $y-(-2) = 2(x-4)$

$c = -10$ $y+2 = 2x-8$

$y = 2x-10$ $y = 2x-10$

4.4

$m_{DE} = 2$

$\therefore m_{GF} = -\dfrac{1}{2}$ [tan $\perp$ radius]

$\dfrac{0-6}{t-8} = -\dfrac{1}{2}$

$-(t-8) = 2(-6)$

$t = 20$

OR

$y = 2x-10$

$0 = 2x-10$

$x = 5$

A(5 ; 0)

In ΔAFG: FA $\perp$ FG

$FA^2 = (6-0)^2 + (8-5)^2 = 45$

$FG^2 = (t-8)^2 + (0-6)^2$

$= t^2 - 16t + 100$

$GA^2 = (t-5)^2$

$= t^2 - 10t + 25$

$\therefore GA^2 = GF^2 + FA^2$

$t^2 - 10t + 25 = t^2 - 16t + 100 + 45$

$6t = 120$

$t = 20$

4.5

$F(8;6)$

G(20 ; 0)

$(8-20)^2+(6-0)^2=r^2$

$r^2=180$

$(x-20)^2+y^2=180$

$x^2+y^2-40x+220=0$

4.6

Smaller circle $r=2\sqrt{5}$

Larger circle $r=6\sqrt{5}$

G(20 ; 0)

$k=20-(6\sqrt{5}-2\sqrt{5})$ or $k=20+(6\sqrt{5}-2\sqrt{5})$

$=20-4\sqrt{5}$ $=20+4\sqrt{5}$

= 11,06 units = 28,94 units

OR

Smaller circle $r=2\sqrt{5}$

$k=2(2\sqrt{5})+20-8\sqrt{5}$ or $k=2(6\sqrt{5})+20-8\sqrt{5}$

$=20-4\sqrt{5}$ $=20+4\sqrt{5}$

= 11,06 units = 28,94 units

OR

$x^2+y^2-40x+220=0$

$y=0$

$\therefore x^2-40x+220=0$

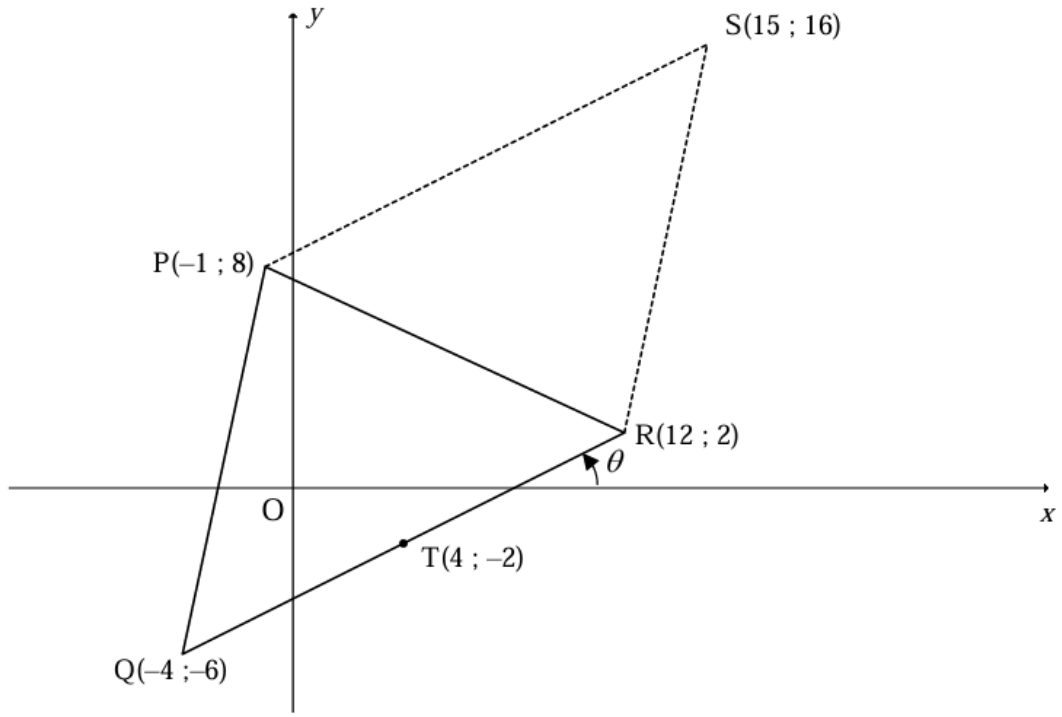
$\therefore x=20+6\sqrt{5}$ or $x=20-6\sqrt{5}$

$\therefore k=20+6\sqrt{5}-\sqrt{20}$ or $k=20-6\sqrt{5}+\sqrt{20}$

$\therefore k=20+4\sqrt{5}$ $\therefore k=20-4\sqrt{5}$

= 11,06 units = 28,94 units

Paper D



3.1	$QR = \sqrt{(-4-12)^2 + (-6-2)^2}$ $= \sqrt{320} = 8\sqrt{5}$ units
3.2	$m_{QR} = \frac{-6-2}{-4-12}$ OR $m_{QR} = \frac{2-(-6)}{12-(-4)}$ $m_{QR} = \frac{1}{2}$ $m_{QR} = \frac{1}{2}$
3.3	$m_{QR} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26,57°$
3.4	$m_{QR} = \frac{1}{2}$ $-6 = \frac{1}{2}(-4) + c$ **OR** $y-2 = \frac{1}{2}(x-12)$ $c = -4$ $y-2 = \frac{1}{2}x-6$ $y = \frac{1}{2}x - 4$ $y = \frac{1}{2}x - 4$
3.5	$Q \rightarrow R : (x\,;y) \rightarrow (x+16\,;y+8)$ $\therefore S(15\,;16)$

3.6

$m_{QR} = \frac{1}{2}$

$m_{PT} = -2$

Equation of PT:

$y = -2x + c$

$8 = -2(-1) + c$

$c = 6$

$y = -2x + 6$

OR

$y - y_1 = -2(x - x_1)$

$y - 8 = -2(x - (-1))$

$y - 8 = -2x - 2$

$y = -2x + 6$

$-2x + 6 = \frac{1}{2}x - 4$

$-4x + 12 = x - 8$

$5x = 20$

$x = 4$

$y = \frac{1}{2}(4) - 4$

$y = -2$

$T(4;-2)$

OR

$PQ = \sqrt{(-4-(-1))^2 + (-6-8)^2} = \sqrt{205}$

$PR = \sqrt{(12-(-1))^2 + (2-8)^2} = \sqrt{205}$

$\therefore$ ΔPQR is isosceles / ΔPQR *is 'n gelykbenige* Δ

$\therefore$ $\perp$ height bisects the base QR /
$\perp$ *hoogte halveer die basis* QR

$\therefore$ T is midpoint of QR / T *is middelpunt van* QR

$\therefore T(4;-2)$

3.7

$PT = \sqrt{(4-(-1))^2 + (-2-8)^2}$

$PT = \sqrt{125} = 5\sqrt{5}$ units = 11,18 units

Area of PQRS = QR.PT

$= (8\sqrt{5})(5\sqrt{5})$

$= 200$ units^2

OR

$$PT = \sqrt{(4-(-1))^2 + (-2-8)^2}$$
$$PT = \sqrt{125} = 5\sqrt{5} \text{ units } = 11{,}18 \text{ units}$$

$$\text{Area of } \Delta PQR = \frac{1}{2}\left(8\sqrt{5}\right)\left(5\sqrt{5}\right)$$
$$= 100 \text{ units}^2$$

$$\text{Area of PQRS} = 2 \times \text{ Area of } \Delta PQR$$
$$= 200 \text{ units}^2$$

4.1	$p = -6$
4.2	$M\hat{D}B = 90°$ [MC ∥ y-axis] $AM = ME = q + 1$ [radii] $MD = q$ $AM^2 = AD^2 + MD^2$ [Pythagoras] $(q+1)^2 = (q-1)^2 + q^2$ $q^2 + 2q + 1 = q^2 - 2q + 1 + q^2$ $q^2 - 4q = 0$ $q(q-4) = 0$ $q \neq 0$ or $q = 4$
4.3	AM = 5 units $(x+6)^2 + (y-4)^2 = 25$
4.4	3 units

4.5

$(x+6)^2+(0-4)^2=25$

$(x+6)^2=9$

$x+6=3$ or $x+6=-3$

$x=-3$ $x=-9$

A $(-9\ ;\ 0)$

B $(-3\ ;\ 0)$

OR

$(x+6)^2+(0-4)^2=25$

$x^2+12x+36+16-25=0$

$x^2+12x+27=0$

$(x+3)(x+9)=0$

$x=-3$ or $x=-9$

A $(-9\ ;\ 0)$

B $(-3\ ;\ 0)$

OR

$q-1=3$

DB = AD = 3 [line from centre ⊥ to chord/ *lyn vanuit midpt ⊥ op koord*]

A $(-9\ ;\ 0)$

B $(-3\ ;\ 0)$

4.6

$m_{MB}=\dfrac{4-0}{-6-(-3)}$

$=-\dfrac{4}{3}$

$m_{BC}=\dfrac{3}{4}$

$y=\dfrac{3}{4}x+c$

$0=\dfrac{3}{4}(-3)+c$

$c=\dfrac{9}{4}$

$y=\dfrac{3}{4}x+\dfrac{9}{4}$

OR

$y-y_1=\dfrac{3}{4}(x+x_1)$

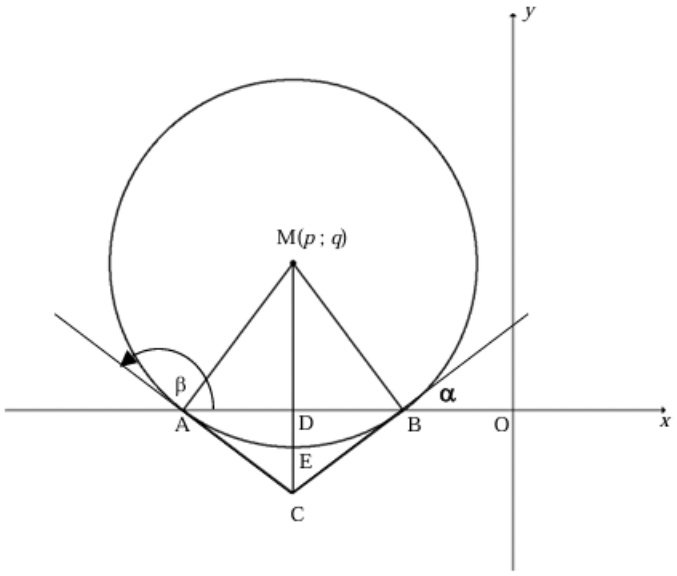
$y-0=\dfrac{3}{4}[x-(-3)]$

$y=\dfrac{3}{4}(x+3)$

$y=\dfrac{3}{4}x+\dfrac{9}{4}$

4.7

$C\left(-6\ ;\ -\dfrac{9}{4}\right)$



4.8

$\tan\alpha = \frac{3}{4}$

$\alpha = 36,87°$

$m_{AC} = -\frac{3}{4}$

$\tan\beta = -\frac{3}{4}$

$\beta = 180° - 36,87°$

$\beta = 143,13°$

$\therefore A\hat{C}B = 106,26°$

OR

$\tan M\hat{A}B = m_{MA} = \frac{4}{3}$

$M\hat{A}B = 53,13°$

$A\hat{M}D = 90° - 53,13°$

$A\hat{M}D = 36,87°$

$M\hat{A}C = 90°$ [tangent $\perp$ radius / *raaklyn⊥radius*]

$A\hat{C}M = 53,13°$

$\therefore A\hat{C}B = 106,26°$ [property of kite / *eienskappe van vlieër*]

OR

$\tan A\hat{C}D = \frac{AD}{DC}$

$\tan A\hat{C}D = \frac{3}{\frac{9}{4}} = \frac{4}{3}$

$A\hat{C}D = 53,13°$

$\therefore A\hat{C}B = 106,26°$ [property of kite/ *eienskappe van vlieër*]

OR

$AB = 6$ units

$AC = BC = \frac{15}{4}$ units [tangent from same point/ *raaklyne vanuit dieselfde punt*]

$(AB)^2 = (AC)^2 + (BC)^2 - 2(AC)(BC)\cos\hat{C}$

$(6)^2 = \left(\frac{15}{4}\right)^2 + \left(\frac{15}{4}\right)^2 - 2\left(\frac{15}{4}\right)\left(\frac{15}{4}\right)\cos\hat{C}$

$\cos A\hat{C}B = -0,28$

$A\hat{C}B = 106,26°$

OR

$\tan M\hat{A}B = m_{MA} = \frac{4}{3}$

$M\hat{A}B = 53,13°$

AMBC is a cyclic quad/ AMBC *is 'n kvh*

$\therefore M\hat{C}B = 53,13°$ [$\angle$s in the same seg/$\angle e$ *in dies segm*]

$\therefore A\hat{C}B = 106,26°$ [property of kite/*eienskappe v vlieër*]

Paper E

3.1	$m_{AB} = \dfrac{3-\frac{1}{2}}{5-0}$ $m_{AB} = \dfrac{1}{2}$
3.2	$m_{CE} = m_{BA} = \dfrac{1}{2}$ $-4 = \dfrac{1}{2}(6) + c$ **OR/*OF*** $y-(-4) = \dfrac{1}{2}(x-6)$ $c = -7$ $y = \dfrac{1}{2}x - 7$
3.3.1	D(0 ; –7) $\dfrac{x_C + 6}{2} = 0$ $\qquad$ $\dfrac{y_C + (-4)}{2} = -7$ $x_C = -6$ $\qquad$ $y_C = -10$ C(–6 ; –10)
3.3.2	$\text{Area } \Delta BCD = \dfrac{1}{2}(7{,}5)(6)$ $= 22{,}5$ $\text{Area } \Delta ABD = \dfrac{1}{2}(7{,}5)(5)$ $= 18{,}75$ $\text{Area ABCD} = 22{,}5 + 18{,}75 = 41{,}25 \text{ units}^2$

3.4.1 K(–6 ; –4)

3.4.2a KC = 6 units; KE = 12 units;

$$CE = \sqrt{(6)^2 + (12)^2} \qquad \text{[Pythagoras]}$$

$$CE = \sqrt{180} = 6\sqrt{5} = 13,42$$

$$\text{Perimeter } \Delta KEC = 6 + 12 + \sqrt{180}$$

$$= 31,42 \text{ units}$$

3.4.2b

$$\tan K\hat{C}E = \frac{KE}{KC} = \frac{12}{6} = 2$$

$$K\hat{C}E = 63,43°$$

OR/*OF*

$$\sin K\hat{C}E = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$$

$$K\hat{C}E = 63,43°$$

OR/*OF*

$$m_{CE} = \frac{1}{2}$$

$$\tan\theta = \frac{1}{2}$$

$$\theta = 26,57°$$

$$K\hat{C}E = 90° - 26,57°$$

$$K\hat{C}E = 63,43°$$

OR

$$KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos K\hat{C}E$$

$$(12)^2 = (6)^2 + \left(\sqrt{180}\right)^2 - 2(6)\left(\sqrt{180}\right)(\cos K\hat{C}E)$$

$$\cos K\hat{C}E = \frac{\sqrt{5}}{5}$$

$$K\hat{C}E = 63{,}43°$$

4.1.1

$$y = x + 1$$
$$b = a + 1$$

4.1.2

$$MR^2 = MK^2$$
$$(a-6)^2 + (b-0)^2 = (a-5)^2 + (b-7)^2$$
$$(a-6)^2 + (a+1)^2 = (a-5)^2 + (a+1-7)^2$$
$$a^2 + 2a + 1 = a^2 - 10a + 25$$
$$12a = 24$$
$$a = 2$$
$$b = 3$$
$$\therefore M(2\ ;\ 3)$$

4.2.1

$$(6-2)^2 + (0-3)^2 = r^2$$
$$r = 5$$

OR/*OF*

$$(2-5)^2 + (3-7)^2 = r^2$$
$$r = 5$$

4.2.2 T(–2 ; 0)
TR = 8 units [line from centre ⊥ to chord]

OR/*OF*

M(2 ; 3)
F(a ; 0)
FR = 4 units
TR = 8 units [line from centre ⊥ to chord]

OR/*OF*

$(x-2)^2+(0-3)^2=25$
$x^2-4x+4+9=25$
$x^2-4x-12=0$
$(x-6)(x+2)=0$
$x=6 \quad \text{or} \quad x=-2$
TR = 8 units

4.3 $m_{\text{radius}}=\frac{7-3}{5-2}$

$m_{\text{radius}}=\frac{4}{3}$

$m_{\text{tangent}}=-\frac{3}{4}$

$7=-\frac{3}{4}(5)+c$ **OR/*OF*** $y-7=-\frac{3}{4}(x-5)$

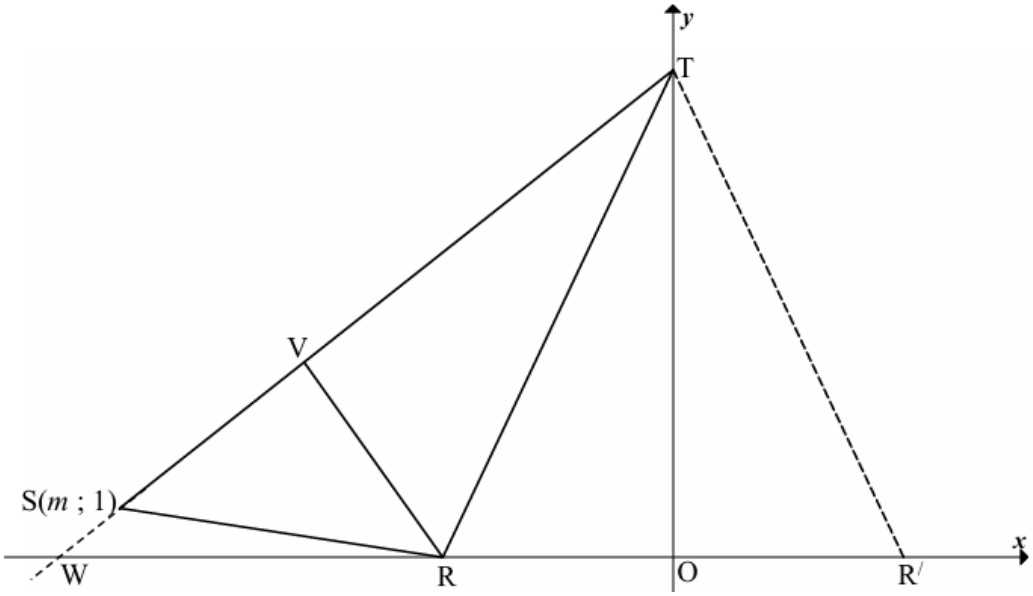
$c=\frac{43}{4}$

$y=-\frac{3}{4}x+\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$

4.4.1 N(2 ; –2)

4.4.2 $(-2-2)^2+(0+2)^2=r^2$
$r^2=20$
$(x-2)^2+(y+2)^2=20$

Paper F



3.1	$2x - y + 10 = 0$ $2x - 0 + 10 = 0$ $x = -5$ R(–5 ; 0)
3.2	R(–5 ; 0) T(0 ; 10) $(RT)^2 = (-5-0)^2 + (0-10)^2$ **OR** $(RT)^2 = 5^2 + 10^2$ (Pythag) $RT = \sqrt{125}$ units **OR/*OF*** $RT = 5\sqrt{5}$ units
3.3	$2RT^2 = 5SR^2$ $2(125) = 5[(m-(-5))^2 + (1-0)^2]$ $5[(m+5)^2 + (1)^2] = 250$ $(m+5)^2 + 1 = 50$ $m^2 + 10m - 24 = 0$ **OR/*OF*** $(m+5)^2 = 49$ $(m-2)(m+12) = 0$ $m+5 = \pm 7$ $m = -5 \pm 7$ $m = 2$ or $m = -12$ $m = 2$ or $m = -12$ N/A N/A $\therefore m = -12$ $\therefore m = -12$

3.4

$$m_{ST} = \frac{1-10}{-12-0}$$

$$m_{ST} = \frac{3}{4}$$

$$\therefore m_{VR} = -\frac{4}{3}$$

$$y = -\frac{4}{3}x + c$$

$$0 = -\frac{4}{3}(-5) + c$$

$$c = -\frac{20}{3}$$

$$y = -\frac{4}{3}x - \frac{20}{3}$$

OR/*OF*

$$y - y_1 = -\frac{4}{3}(x - x_1)$$

$$y - 0 = -\frac{4}{3}(x - (-5))$$

$$y = -\frac{4}{3}(x + 5)$$

$$y = -\frac{4}{3}x - \frac{20}{3}$$

3.5

VR: $y = -\frac{4}{3}x - \frac{20}{3}$

ST: $y = \frac{3}{4}x + 10$

$$\frac{3}{4}x + 10 = -\frac{4}{3}x - \frac{20}{3}$$

$$9x + 120 = -16x - 80$$

$$25x = -200$$

$$x = -8$$

$$y = 4$$

$$\therefore V(-8\ ;\ 4)$$

3.6 $R'(5;0)$

$$VR = \sqrt{[-8-(-5)]^2 + (4-0)^2} = 5$$

$$VT = \sqrt{(-8-0)^2 + (4-10)^2} = 10$$

$$\text{Area of } RVTR' = \frac{1}{2}(VR)(VT) + \frac{1}{2}(RR')(OT)$$

$$= \frac{1}{2}(5)(10) + \frac{1}{2}(10)(10)$$

$$= 25 + 50$$

$$= 75 \text{ units}^2$$

OR/*OF*

ST: $y = \frac{3}{4}x + 10$

$$0 = \frac{3}{4}x + 10$$

$$x = -\frac{40}{3} \text{ or } -13\frac{1}{3}$$

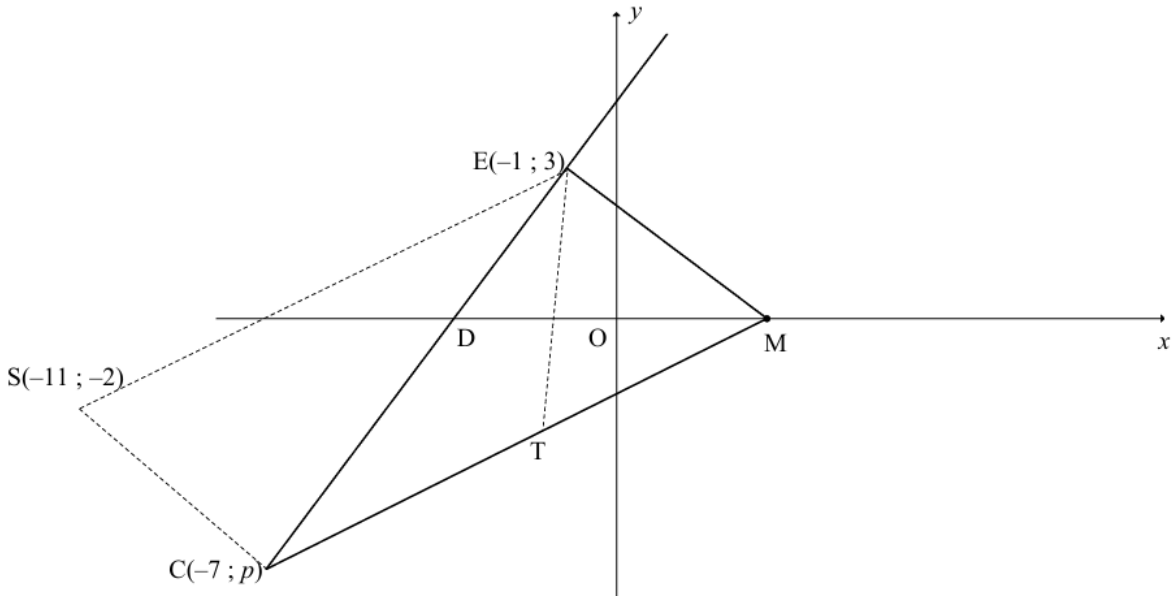
$$W\left(-\frac{40}{3};0\right)$$

$$WR' = 5 - \left(-\frac{40}{3}\right) = \frac{55}{3} = 18\frac{1}{3}$$

$$\text{Area of } RVTR' = \text{Area of } \Delta TWR' - \text{Area of } \Delta WVR$$

$$= \frac{1}{2}\left(5 + \frac{40}{3}\right)(10) - \frac{1}{2}\left(-5 + \frac{40}{3}\right)(4)$$

$$= 75 \text{ units}^2$$



4.1	$C\hat{E}M = 90°$
4.2	$m_{ME} = \frac{0-3}{3-(-1)}$
	$m_{ME} = -\frac{3}{4}$
	$\therefore m_{ED} = \frac{4}{3}$
	$3 = \frac{4}{3}(-1) + c$ **OR/*OF*** $y - 3 = \frac{4}{3}(x-(-1))$
	$y = \frac{4}{3}x + \frac{13}{3}$ $y = \frac{4}{3}x + \frac{13}{3}$
	OR/*OF*
	$DM = \sqrt{(5)^2 + \left(\frac{15}{4}\right)^2}$ [Pythagoras]
	$DM = \frac{25}{4}$ or 6,25 units
	$\therefore D\left(-\frac{13}{4};0\right)$
	$m_{ED} = \frac{3-0}{-1-\left(-\frac{13}{4}\right)}$
	$\therefore m_{ED} = \frac{4}{3}$
	$3 = \frac{4}{3}(-1) + c$ **OR/*OF*** $y - 3 = \frac{4}{3}(x-(-1))$
	$y = \frac{4}{3}x + \frac{13}{3}$ $y = \frac{4}{3}x + \frac{13}{3}$

4.3

$$y = \frac{4}{3}x + \frac{13}{3}$$

$$0 = \frac{4}{3}x + \frac{13}{3}$$

$$x_D = -\frac{13}{4}$$

$$\therefore \text{DM} = 3 - \left(-\frac{13}{4}\right)$$

$$\text{DM} = \frac{25}{4} \text{ or } 6{,}25 \text{ units}$$

OR/*OF*

$$\text{EM} = 5 \text{ units}$$

$$\text{ED} = \frac{15}{4} \text{ units}$$

$$\text{DM} = \sqrt{(5)^2 + \left(\frac{15}{4}\right)^2} \qquad \text{[Pythagoras]}$$

$$\text{DM} = \frac{25}{4} \text{ or } 6{,}25 \text{ units}$$

4.4

EC: $y = \frac{4}{3}x + \frac{13}{3}$

$$p = \frac{4}{3}(-7) + \frac{13}{3}$$

$$p = -5$$

OR

$$m_{EC} = \frac{4}{3}$$

$$\frac{p-3}{-7-(-1)} = \frac{4}{3}$$

$$p - 3 = \frac{4}{3}(-6)$$

$$p = -5$$

4.5

$M \rightarrow E: (x\ ;\ y) \rightarrow (x - 4\ ;\ y + 3)$ [translation]

$C \rightarrow S: (-7\ ;\ -5) \rightarrow (-7 - 4\ ;\ -5 + 3)$

$\therefore S(-11\ ;\ -2)$

OR/*OF*

$M \rightarrow C: (x\ ;\ y) \rightarrow (x - 10\ ;\ y - 5)$ [translation]

$E \rightarrow S: (-1\ ;\ 3) \rightarrow (-1 - 10\ ;\ 3 - 5)$

$\therefore S(-11\ ;\ -2)$

OR/*OF*

$E(-1\ ;\ 3)$ and $C(-7\ ;\ -5)$

$\left(\frac{-1+(-7)}{2}\ ;\ \frac{3+(-5)}{2}\right)$ [Midpoint of EC]

$= (-4; -1)$

$S(x\ ;\ y)$ and $M(3\ ;\ 0)$

$\frac{x_S + 3}{2} = -4 \qquad \frac{y_S + 0}{2} = -1$

$x_S = -11 \qquad y_S = -2$

$\therefore S(-11\ ;\ -2)$

4.6

$r_{NEW} = 5 + 7$

$r_{NEW} = 12$

$MS = \sqrt{(3-(-11))^2 + (0-(-2))^2}$

MS = $\sqrt{200}$ or $10\sqrt{2}$ or 14,14 units

14,14 > 12

$\therefore S(-11; -2)$ lies outside the circle

4.7

Inclination of EM: $\tan \hat{M} = m_{ME} = -\frac{3}{4}$

ref. $\angle = 36{,}87°$

Inclination of EM = $143{,}13°$

$\therefore E\hat{M}D = 36{,}87°$

Inclination of CM: $\tan \hat{M} = m_{CM} = \frac{5}{10}$ or $\frac{1}{2}$

$\therefore \hat{M} = 26{,}57°$

$\therefore E\hat{M}T = 26{,}57° + 36{,}87°$

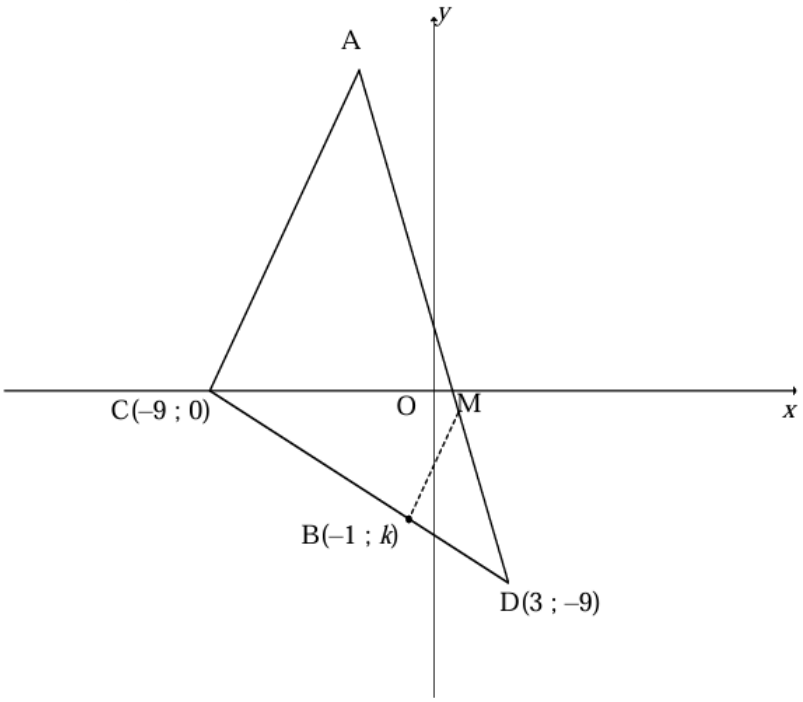
$= 63{,}44°$

But EM = MT [radii]

$\therefore E\hat{T}M = \frac{180° - 63{,}44°}{2}$

$\therefore E\hat{T}M = 58{,}28°$

Paper G



3.1	$m_{DC}=\dfrac{-9-0}{3-(-9)}$ **OR/*OF*** $m_{DC}=\dfrac{0-(-9)}{-9-3}$ $m_{DC}=-\dfrac{3}{4}$ $m_{DC}=-\dfrac{3}{4}$
3.2	Equation of DC: $0=-\dfrac{3}{4}(-9)+c$ **OR/*OF*** $y-0=-\dfrac{3}{4}(x-(-9))$ $c=\dfrac{-27}{4}$ or $-6\dfrac{3}{4}$ $y=-\dfrac{3}{4}(x+9)$ $y=-\dfrac{3}{4}x-\dfrac{27}{4}$ $y=-\dfrac{3}{4}x-\dfrac{27}{4}$
3.3	$k=-\dfrac{3}{4}(-1)-\dfrac{27}{4}$ **OR/*OF*** $\dfrac{k-(-9)}{-1-3}=\dfrac{-3}{4}$ **OR/*OF*** $\dfrac{k-0}{-1-(-9)}=\dfrac{-3}{4}$ $k=\dfrac{3}{4}-\dfrac{27}{4}$ **OR/*OF*** $k+9=3$ **OR/*OF*** $k=-\dfrac{3}{4}(8)$ $k=-6$ $k=-6$ $k=-6$
3.4	$DC=\sqrt{(3+9)^2+(-9-0)^2}$ DC = 15 units

3.5

$$DB = \sqrt{(3-(-1))^2 + (-9-(-6))^2}$$

$$DB = 5$$

$$\therefore \frac{DB}{DC} = \frac{5}{15} = \frac{1}{3}$$

3.6

$$\frac{DM}{DA} = \frac{DB}{DC} = \frac{1}{3}$$

$$\frac{\text{Area } \Delta MBD}{\text{Area } \Delta ACD} = \frac{\frac{1}{2}(DM)(DB)\left(\sin \hat{D}\right)}{\frac{1}{2}(DA)(DC)\left(\sin \hat{D}\right)}$$

$$= \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{1}{9}$$

3.7

$y = -4x + c$

$m_{AD} = -4$

$-9 = -4(3) + c$ **OR/*OF*** $\frac{y+9}{x-3} = -4$

$c = 3$ $y + 9 = -4x + 12$

$y = -4x + 3$ $y = -4x + 3$

$(x-3)^2 + (y+9)^2 = 612$

$(x-3)^2 + (-4x+3+9)^2 = \left(\sqrt{612}\right)^2$

$(x-3)^2 + (-4x+12)^2 = 612$

$x^2 - 6x + 9 + 16x^2 - 96x + 144 = 612$

$17x^2 - 102x - 459 = 0$

$x^2 - 6x - 27 = 0$

$(x-9)(x+3) = 0$

$x = 9$ or $x = -3$

N/A

$y = -4(-3) + 3$

$y = 15$

$A(-3;15)$

OR

$-9 = -4(3) + c$

$c = 3$

$y = -4x + 3$

N(0 ; 3)

$\text{ND} = \sqrt{(3-0)^2 + (-9-3)^2}$

$= 3\sqrt{17}$

$\text{AD} = 6\sqrt{17}$

$\text{ND} = \frac{1}{2}\text{AD}$

N is the midpoint of AD

A(–3 ; 15)

4.1	L(–1 ; –3)
4.2	$m_{MP} = \frac{9-3}{3-1}$ $m_{MP} = 3$ $m_{ST} = -\frac{1}{3}$ $9 = -\frac{1}{3}(3) + c$ $\quad$ $y - 9 = -\frac{1}{3}(x-3)$ $c = 10$ $\quad$ **OR/*OF*** $\quad$ $y - 9 = -\frac{1}{3}x + 1$ $y = -\frac{1}{3}x + 10$ $\quad$ $y = -\frac{1}{3}x + 10$
4.3	$(x-1)^2 + (y-3)^2 = r^2$ $(3-1)^2 + (9-3)^2 = r^2$ $r^2 = 40$ $(x-1)^2 + (y-3)^2 = 40$ $x^2 - 2x + 1 + y^2 - 6y + 9 = 40$ $x^2 + y^2 - 2x - 6y - 30 = 0$

4.4

$d^2+(1)^2-2d-6(1)-30=0$

$d^2-2d-35=0$

$(d-7)(d+5)=0$

$d=7 \text{ or } d=-5$

$\therefore d=7$

OR/*OF*

$(x-1)^2+(y-3)^2=40$

$(d-1)^2+(1-3)^2=40$

$(d-1)^2=36$

$d-1=6 \text{ or } d-1=-6$

$d=7 \text{ or } d=-5$

$\therefore d=7$

OR/*OF*

$P\hat{R}L=90°$ ($\angle$ in semi-circle)

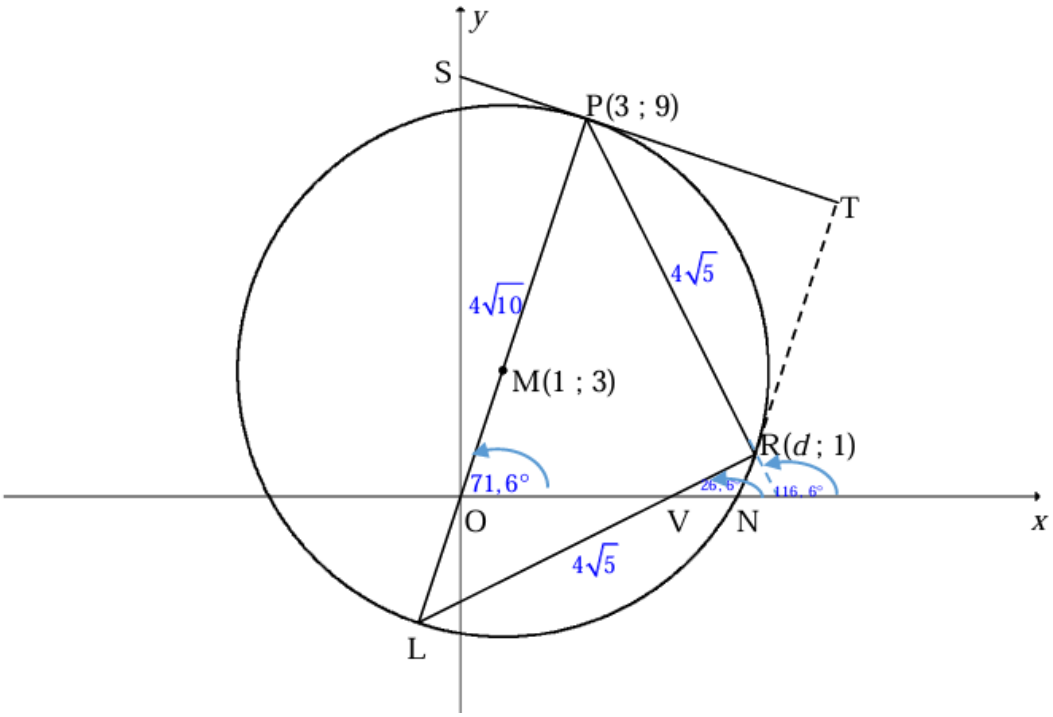
$\frac{9-1}{3-d}\times\frac{1-(-3)}{d-(-1)}=-1$

$d^2-2d-35=0$

$(d-7)(d+5)=0$

$d=7 \text{ or } d=-5$

$\therefore d=7$


y
S
P(3 ; 9)
T
$4\sqrt{5}$
$4\sqrt{10}$
M(1 ; 3)
R(d ; 1)
71,6°
26,6°
116,6°
O
V
N
x
$4\sqrt{5}$
L

4.5

$m_{PO} = 3$

$\therefore \tan P\hat{O}V = 3$

$P\hat{O}V = 71{,}565...°$

$$m_{RL} = \frac{1-(-3)}{7-(-1)}$$

$$= \frac{1}{2}$$

$$\therefore \tan R\hat{V}N = \frac{1}{2}$$

$R\hat{V}N = 26{,}565...°$

$\hat{L} = 71{,}565...° - 26.565...°$ [ext. $\angle$ of Δ / *buite* $\angle$ *van* Δ]

$\hat{L} = 45°$

OR/*OF*

$\hat{R} = 90°$ [$\angle$ in semi-circle / $\angle$ *in 'n halwe sirkel*]

$PR^2 = (3-7)^2 + (9-1)^2$

$PR = \sqrt{80} = 4\sqrt{5}$ units

$PL^2 = (3-(-1))^2 + (9-(-3))^2$ **OR** $RL^2 = (7+1)^2 + (1+3)^2$

$PL = \sqrt{160} = 4\sqrt{10}$ $RL = \sqrt{80} = 4\sqrt{5}$

$\sin \hat{L} = \frac{4\sqrt{5}}{4\sqrt{10}}$ **OR** $\cos \hat{L} = \frac{4\sqrt{5}}{4\sqrt{10}}$ **OR** $\tan \hat{L} = \frac{4\sqrt{5}}{4\sqrt{5}}$

$\hat{L} = 45°$

OR/*OF*

$PL = \sqrt{(3+1)^2 + (9+3)^2} = \sqrt{160} = 4\sqrt{10}$

$PR = \sqrt{(7-3)^2 + (1-9)^2} = \sqrt{80} = 4\sqrt{5}$

$LR = \sqrt{(7+1)^2 + (1+3)^2} = \sqrt{80} = 4\sqrt{5}$

$\cos L = \dfrac{80+160-80}{2\sqrt{80} \times \sqrt{160}}$

$\cos L = \dfrac{\sqrt{2}}{2}$

$\hat{L} = 45°$

4.6

$m_{RM} = \dfrac{1-3}{7-1}$

$= -\dfrac{1}{3}$

$m_{RT} = 3$ (tan ⊥ rad)

$m_{PT} = -\dfrac{1}{3}$

$m_{RT} \times m_{PT} = -1$

$PT \perp RT$

OR/*OF*

$m_{MR} = \dfrac{3-1}{1-7}$

$= -\dfrac{1}{3}$

$m_{PT} = -\dfrac{1}{3}$ [proved in Q4.2]

$m_{PT} = m_{MR}$

$\therefore PT \parallel MR$

$M\hat{R}T = 90°$ [radius ⊥ tangent / *raaklyn ⊥ radius*]

$P\hat{T}R = 90°$ [co-int ∠s; PT ∥ MR/*ooreenkomst. ∠e*; PT ∥ MR]

$PT \perp RT$

OR/*OF*

$T\hat{P}R = \hat{L} = 45°$ [tan-chord theorem/ *∠ tussen raaklyn en koord*]

$TP = TR$ [tans from common pt]

$\therefore T\hat{P}R = T\hat{R}P = 45°$ [∠s opp equal sides/ *∠e teenoor gelyke sye*]

$\therefore P\hat{T}R = 90°$ [sum of ∠s in Δ / *binne ∠e van Δ*]

$PT \perp RT$

Paper H

3.1	$M\left(\frac{4+8}{2};\frac{-8+0}{2}\right)$ $M(6;-4)$
3.2	$m_{NS}=\frac{0-(-16)}{8-0}$ or $m_{NQ}=\frac{0-(-8)}{8-4}$ or $m_{QS}=\frac{-8-(-16)}{4-0}$ $=2$
3.3	$m_{LQ}\times 2=-1$ $\quad$ [LQ ⊥ NS] $\therefore m_{LQ}=-\frac{1}{2}$ $-8=-\frac{1}{2}(4)+c$ **OR** $y+8=-\frac{1}{2}(x-4)$ $c=-6$ $\quad$ $y+8=-\frac{1}{2}x+2$ $\therefore y=-\frac{1}{2}x-6$
3.4	OS is the radius of circle passing through S $(x-0)^2+(y-0)^2=(16)^2$ $x^2+y^2=256$
3.5	$m_{RM}=m_{LQ}=-\frac{1}{2}$ $\quad$ [RM ∥ LQ] $-4=-\frac{1}{2}(6)+c$ **OR** $y+4=-\frac{1}{2}(x-6)$ $c=-1$ $\quad$ $y+4=-\frac{1}{2}x+3$ $\therefore y=-\frac{1}{2}x-1$ $T(0;-1)$

3.6 $T(0;-1)$, $P(0;-6)$ and $S(0;-16)$

$\therefore PS = 10$ units and $TS = 15$ units

$$\frac{LS}{RS} = \frac{PS}{TS} = \frac{2}{3}$$

[prop theorem; RM || LP]
OR [line || one side of Δ/*lyn* || *een sy v* Δ]

OR

M(6 ; –4), Q(4 ; –8) and S(0 ; –16)

$MS = \sqrt{180} = 6\sqrt{5}$ and $QS = \sqrt{80} = 4\sqrt{5}$

$$\frac{LS}{RS} = \frac{QS}{MS} = \frac{2}{3}$$

[prop theorem; RM || LQ]
OR [line || one side of Δ/*lyn* || *een sy v* Δ]

3.7 area of PTMQ = area of ΔTSM – area of ΔPSQ

$$= \frac{1}{2}.ST.\perp h_M - \frac{1}{2}.PS.\perp h_Q$$

$$= \frac{1}{2}(15)(6) - \frac{1}{2}(10)(4)$$

$$= 45 - 20$$

$$= 25 \text{ square units}$$

OR

$$TM = \sqrt{45} = 3\sqrt{5} = 6{,}71$$

$$MQ = \sqrt{20} = 2\sqrt{5} = 4{,}47$$

$$PQ = \sqrt{20} = 2\sqrt{5} = 4{,}47$$

$$\text{area of trapezium PTMQ} = \frac{1}{2}\left(3\sqrt{5} + 2\sqrt{5}\right)\left(2\sqrt{5}\right)$$

$$= \frac{1}{2}\left(5\sqrt{5}\right)\left(2\sqrt{5}\right)$$

$$= 25 \text{ square units}$$

OR

$MQ = \sqrt{20} = 2\sqrt{5}$

$PQ = \sqrt{20} = 2\sqrt{5}$

$TP = 5$

area of PTMQ= area of ΔMTP+ area of ΔPQM

$$\text{area of PTMQ} = \frac{1}{2}\text{TP} \times \perp h_M + \frac{1}{2}\text{MQ} \times \text{PQ}$$

area of PTMQ$= 10 + 15 = 25$

4.1

$PV = r = \sqrt{10}$

$PV = \sqrt{(k-(-3))^2 + (1-4)^2} = \sqrt{10}$

$(PV)^2 = (k-(-3))^2 + (1-4)^2 = 10$

$k^2 + 6k + 9 + 9 = 10$

$k^2 + 6k + 8 = 0$

$(k+4)(k+2) = 0$

$k = -4 \quad \text{or} \quad k = -2$

$\therefore k = -2$

OR

$(k+3)^2 + 9 = 10$

$(k+3)^2 = 1$

$k + 3 = 1 \text{ or } k + 3 = -1$

4.2

$x^2 + 6x + y^2 - 8y + 15 = 0$

y-intercepts: $(0)^2 + 6(0) + y^2 - 8y + 15 = 0$

$(y-3)(y-5) = 0$

$y_C = 3 \quad \text{or} \quad y_B = 5$

$\therefore \text{BC} = 2 \text{ units}$

4.3.1

$$m_{TC} = \frac{3-1}{0-(-2)}$$

$$= 1$$

$$\tan\alpha = 1$$

$$\therefore \alpha = 45°$$

OR

$$y = mx + 3$$

$$1 = m(-2) + 3$$

$$m_{TC} = 1$$

$$\tan\alpha = 1$$

$$\therefore \alpha = 45°$$

4.3.2

$\hat{BCV} = 135°$ [ext ∠ of Δ/*buite* ∠ v Δ]

$\therefore \hat{VWB} = 45°$ [opp ∠s of cyclic quad/*teenoorst. ∠e v kvh*]

OR

$\hat{TCO} = 45°$ [∠s of Δ/*∠e v* Δ]

$\therefore \hat{VWB} = 45°$ [ext ∠s of cyclic quad/*buite ∠ v kvh*]

4.4.1 $Q(-3;-2)$

4.4.2 $(x+3)^2 + (y+2)^2 = 10$

4.4.3 $x = -2$ or $x = -4$

Paper I

3.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ $= \frac{1}{4}$	**OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$	
3.2	$y = mx + c$ $0 = \frac{1}{4}(12) + c$ $c = -3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ $-2 = \frac{1}{4}(4) + c$ $c = -3$ $y = \frac{1}{4}x - 3$	or $y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{4}(x - 12)$ $y = \frac{1}{4}x - 3$ or $y - y_1 = m(x - x_1)$ $y - (-2) = \frac{1}{4}(x - 4)$ $y = \frac{1}{4}x - 3$

3.3.1

$$y=\frac{1}{4}x-3$$

$$k=\frac{1}{4}k-3$$

$$\frac{3}{4}k=-3$$

$$k=-4$$

$$\therefore \text{B}(-4;-4)$$

OR/*OF*

$$m_{\text{BE}}=\frac{1}{4}$$

$$\frac{0-k}{12-k}=\frac{1}{4}$$

$$-4k=12-k$$

$$k=-4$$

$$\therefore \text{B}(-4;-4)$$

OR/*OF*

$$m_{\text{BE}}=\frac{1}{4}$$

$$\frac{k}{k-12}=\frac{1}{4}$$

$$4k=k-12$$

$$k=-4$$

OR/*OF*

$$m_{\text{AB}}=\tan 81{,}87^\circ$$

$$m_{\text{AB}}=7$$

$$m_{\text{AB}}=\frac{10-k}{-2-k}$$

$$7(-2-k)=10-k$$

$$-14-7k=10-k$$

$$-6k=24$$

$$k=-4$$

$$\therefore \text{B}(-4;-4)$$

OR/*OF*

EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$

$$\frac{1}{4}x-3=7x+24$$

$$\frac{27}{4}x=-27$$

$$x=k=-4$$

$$\therefore \text{B}(-4;-4)$$

3.3.2

In ΔAFG:

$$m_{AC} = \frac{10-(-2)}{-2-4} = -2$$

$$\tan\theta = m_{AC} = -2$$

$$\theta = 180° - 63,43...°$$

$$\therefore \theta = 116,57°$$

$$\therefore \hat{A} = 116,57° - 81,87° \text{ [ext } \angle \text{ of } \Delta \text{]}$$

$$\therefore \hat{A} = 34,70°$$

OR/*OF*

In ΔABC:

$$a = BC = 2\sqrt{17};\ b = AC = 6\sqrt{5};\ c = AB = 10\sqrt{2}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$$

$$\cos A = \frac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$$

$$= 0,822...$$

$$\therefore A = 34,7°$$

3.3.3

$$M\left(\frac{12+(-2)}{2}; \frac{10+(0)}{2}\right)$$

Diagonals intersect at the point (5 ; 5)

3.4.1

$$BE = ET$$

$$4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$$

$$\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$$

$$272 = 144 - 24p + p^2 + p^2$$

$$p^2 - 12p - 64 = 0$$

$$(p-16)(p+4) = 0$$

$$\therefore p = 16 \quad \text{or} \quad p = -4 \text{ (n.a.)}$$

$$\therefore T(16;\ 16)$$

3.4.2a

$$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$$

3.4.2b

$$m_{radius} = \frac{1}{4}$$

$$m_{tangent} = -4$$

$$y = -4x + c$$

$$-4 = -4(-4) + c$$

$$c = -20$$

$$y = -4x - 20$$

OR/*OF*

$$y - y_1 = -4(x - x_1)$$

$$y - (-4) = -4(x - (-4))$$

$$y = -4x - 20$$

4.1	Radius = 4 units/*eenhede*
4.2.1	$CD \perp CN$ $\therefore C(-1\,;\,7)$
4.2.2	CD = 6 units $\therefore D(5\,;\,7)$
4.2.3	$\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}[\text{Area of } \parallel^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$

OR/*OF*

Let angle of inclination of BC = α

$\tan\alpha = \frac{5}{3}$

$\alpha = 59{,}036...°$

$B\hat{C}D = 180° - \alpha$

$B\hat{C}D = 180° - 59{,}036...°$

$B\hat{C}D = 120{,}96°$

$\text{Area } \Delta BCD = \frac{1}{2}\left(\sqrt{34}\right)(6)\sin 120{,}96°$

$= 15 \text{ units}^2$

4.3.1 M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$]

$\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$

$MN = \sqrt{32} = 4\sqrt{2} = 5{,}66 \text{ units}$

4.3.2 M(3 ; –1)

$m_{MN} = \frac{3-(-1)}{-1-3} = -1$

MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$

$c = 2$ $y - 3 = -x - 1$

$\therefore y = -x + 2$ $y = -x + 2$

$x = -x + 2$

$2x = 2$

$x = 1$

$\therefore y = 1$

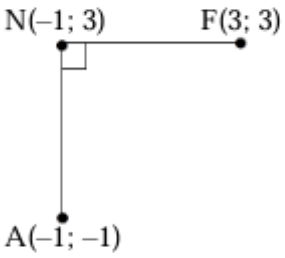
midpoint (1 ; 1)

OR/*OF*

N(–1 ; 3)

$y_F = y_N = 3$

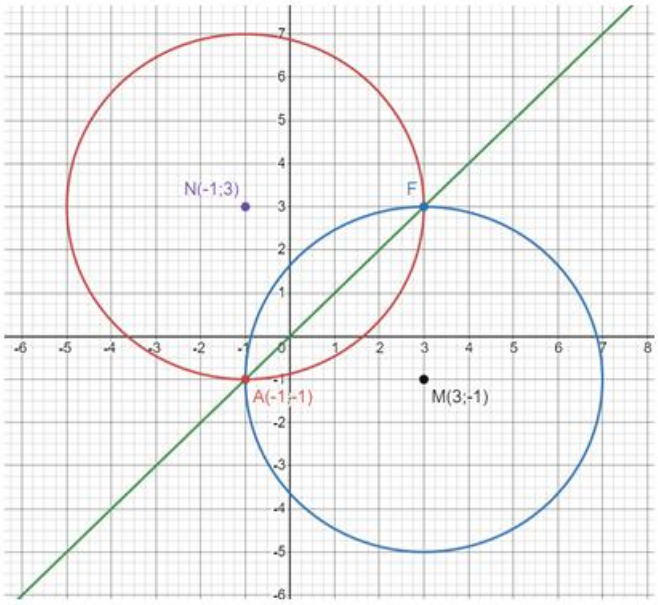
Reflected about $y = x$

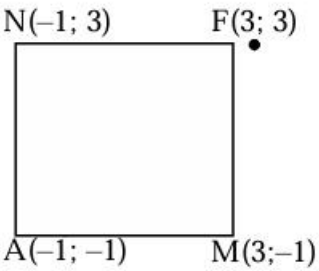


$\therefore$ F(3 ; 3)

$\text{midpoint}\left(\frac{-1+3}{2}; \frac{-1+3}{2}\right) = (1 ; 1)$

OR/*OF*



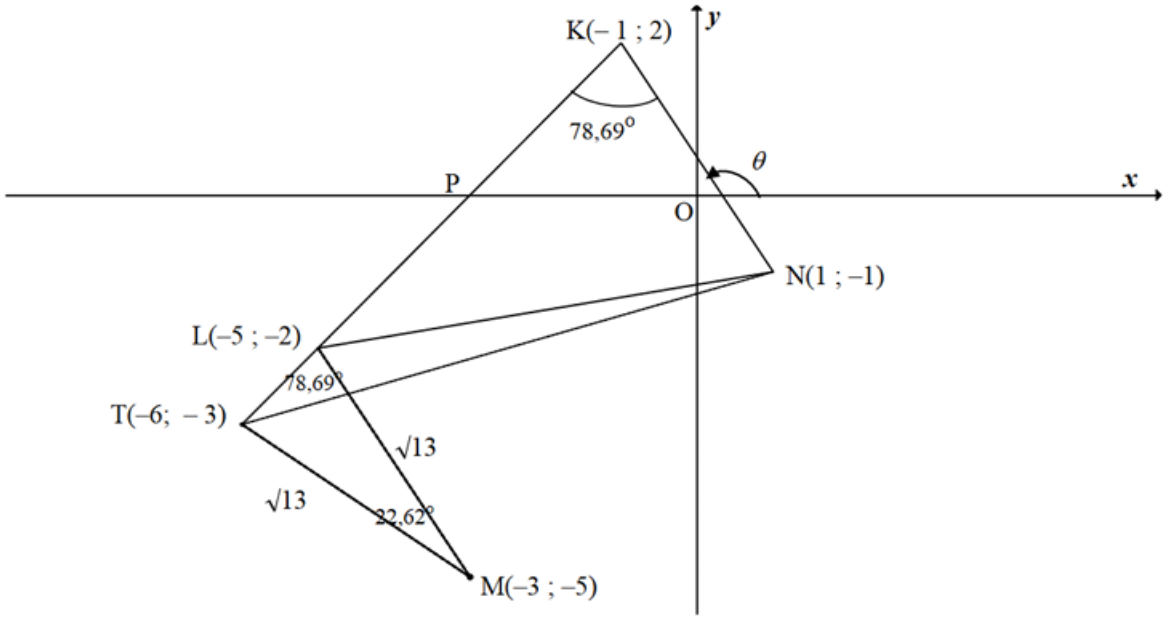


NAMF is a square (NA=NF=AM=MF and NA ⊥ AM)

Midpoint NM = (1 ; 1)

= Midpoint of AF

Paper J



3.1.1	$m_{KN} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \dfrac{2-(-1)}{-1-1}$ $= -\dfrac{3}{2}$
3.1.2	$\tan\theta = m_{KN} = -\dfrac{3}{2}$ $\theta = 180° - 56{,}31°$ $\theta = 123{,}69°$
3.2	Inclination KL = $123{,}69° - 78{,}69° = 45°$ [ext $\angle\Delta$] $\tan 45° = m_{KL} = 1$
3.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y = x + 3$
3.4	$KN = \sqrt{(1+1)^2 + (-1-2)^2}$ $KN = \sqrt{13}$ or 3,61

3.5.1

$(x+3)^2+(y+5)^2=13 \quad ...(1)$

L is a point on KL

$y=x+3 \quad ...(2)$

(2) in (1):

$(x+3)^2+(x+3+5)^2=13$

$x^2+6x+9+x^2+16x+64=13$

$2x^2+22x+60=0$

$x^2+11x+30=0$

$(x+5)(x+6)=0$

$x=-5$ or $x=-6$

$y=-2$ or $y=-3$

L(–5 ; –2) or (–6 ; –3)

OR/OF

$(x+3)^2+(y+5)^2=13 \quad ...(1)$

L is a point on KL

$y=x+3 \quad \therefore x=y-3 \quad ...(2)$

(2) in (1):

$(y-3+3)^2+(y+5)^2=13$

$y^2+y^2+10y+25=13$

$2y^2+10y+12=0$

$y^2+5y+6=0$

$(y+2)(y+3)=0$

$y=-2$ or $y=-3$

$x=-5$ or $x=-6$

L(–5 ; –2) or (–6 ; –3)

3.5.2

Midpoint of KM: (– 2 ; – 1,5)

$\therefore \frac{x_L+1}{2}=-2$ and $\frac{y_L-1}{2}=-\frac{3}{2}$

$\therefore$ L(– 5 ; – 2)

OR/*OF*

$m_{KN}=m_{LM}$

$\frac{y-(-5)}{x-(-3)}=-\frac{3}{2}$

$2(x+3+5)=-3(x+3)$

$2x+16=-3x-9$

$5x=-25$

$x=-5$

$\therefore$ L(– 5 ; – 2)

OR/*OF*

N→M:		N→K:
$(x\,;y) \rightarrow (x-4\,;y-4)$		$(x\,;y) \rightarrow (x-2\,;y+3)$
$\therefore$ L$(-1-4\,;2-4)$	**OR/OF**	$\therefore$ L$(-3-2\,;-5+3)$
$\therefore$ L$(-5\,;-2)$		$\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M:		N→K:
$(x\,;y) \rightarrow (x-4\,;y-4)$		$(x\,;y) \rightarrow (x-2\,;y+3)$
$\therefore$ L$(-1-4\,;2-4)$	**OR/OF**	$\therefore$ L$(-3-2\,;-5+3)$
$\therefore$ L$(-5\,;-2)$		$\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M:		N→K:
$(x\,;y) \rightarrow (x-4\,;y-4)$		$(x\,;y) \rightarrow (x-2\,;y+3)$
$\therefore$ L$(-1-4\,;2-4)$	**OR/OF**	$\therefore$ L$(-3-2\,;-5+3)$
$\therefore$ L$(-5\,;-2)$		$\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M:		N→K:
$(x\,;y) \rightarrow (x-4\,;y-4)$		$(x\,;y) \rightarrow (x-2\,;y+3)$
$\therefore$ L$(-1-4\,;2-4)$	**OR/OF**	$\therefore$ L$(-3-2\,;-5+3)$
$\therefore$ L$(-5\,;-2)$		$\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M:		N→K:
$(x\,;y) \rightarrow (x-4\,;y-4)$		$(x\,;y) \rightarrow (x-2\,;y+3)$
$\therefore$ L$(-1-4\,;2-4)$	**OR/OF**	$\therefore$ L$(-3-2\,;-5+3)$
$\therefore$ L$(-5\,;-2)$		$\therefore$ L$(-5\,;-2)$

3.6 T(–6 ; –3) (from Question 3.5.1)

$$KT = \sqrt{(-1-(-6))^2 + (2-(-3))^2}$$

$$= \sqrt{50}$$

$KN = \sqrt{13}$ (CA from 3.4)

$$\text{Area of } \Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$$

$$= \frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$$

$$= 12{,}50 \text{ square units}$$

OR/*OF*

In ΔKLM:

$$\frac{TL}{\sin 22{,}62°} = \frac{\sqrt{13}}{\sin 78{,}69°}$$

$$TL = 1{,}414..$$

$$KL = \sqrt{(-1-(-5))^2 + (2-(-2))^2}$$

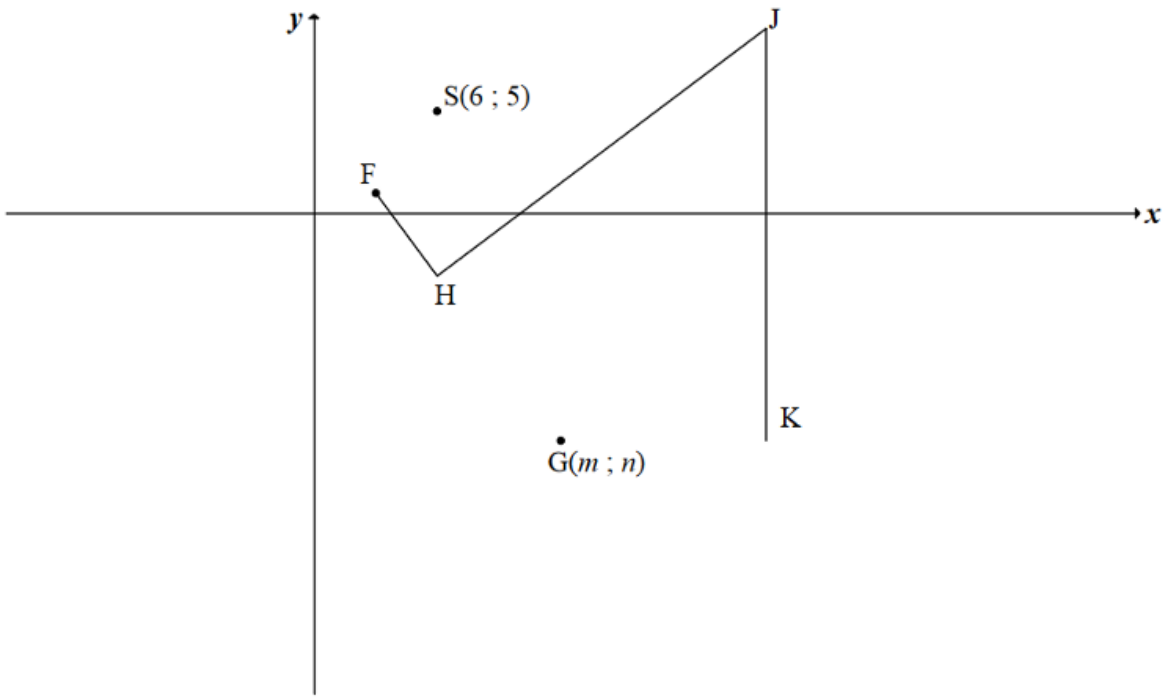
$$= \sqrt{32}$$

$\therefore$ KT = 7,0708…

$$\text{Area of } \Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$$

$$= \frac{1}{2}(7{,}0708).\sqrt{13}\sin 78{,}69°$$

$$= 12{,}50 \text{ square units}$$



4.1	$F(3;1)$
4.2	$FS = \sqrt{(6-3)^2 + (5-1)^2}$ $FS = 5$
4.3	$FH(FS) : HG = 1 : 2$ $\therefore HG = 2\ FH$ $= 10$
4.4	Tangents from common/same point / *Raaklyne vanaf gemeenskaplike of dieselfde punt*
4.5.1	$F\hat{H}J = 90°$ [tan $\perp$ radius / *rkl* $\perp$ *radius*] $FJ^2 = 20^2 + 5^2$ [Pyth theorem/*stelling*] $FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62
4.5.2	$(x-m)^2 + (y-n)^2 = 100$

4.5.3

K(22; n) [radius ⊥ tangent]

GK = HG = 10 [radii]

FH = FS = 5 [radii]

$m = 22 - 10$

$m = 12$

F, H and G are collinear [HJ is a common tangent]

F, H en G is saamlynig *[HJ is 'n gemeemskaplike raaklyn]*

$FG^2 = (12-3)^2 + (n-1)^2$

$15^2 = 81 + (n-1)^2$

$(n-1)^2 = 144$

$n - 1 = \pm 12$

$n \neq 13$ or $n = -11$

$\therefore$ G(12; –11)

OR/*OF*

$n^2 - 2n - 143 = 0$

$(n+11)(n-13) = 0$

$n = -11$ or $n \neq 13$

OR/*OF*

K(22; n) [radius ⊥ tangent]

GK = HG = 10 [radii]

FH = FS = 5 [radii]

$m = 22 - 10$

$m = 12$

Let J(22 ; y):

$FJ^2 = (22-3)^2 + (y-1)^2$

$425 = 361 + y^2 - 2y + 1$

$0 = y^2 - 2y - 63$

$0 = (y-9)(y+7)$

$\therefore y = 9$ or/of $y \neq -7$

$\therefore n = 9 - 20 = -11$

$\therefore$ G(12; –11)

TRIGONOMETRY

Paper A

5.1.1	$\sin(360°+x)$ $=\sin x$
5.1.2	$x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(-3)^2}$ $=-2$ $\tan x=\dfrac{-3}{-2}$ $=\dfrac{3}{2}$ **OR/*OF*** $x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(3)^2}$ $=2$ $\tan x=\dfrac{3}{2}$
5.1.3	$\cos(180°+x)$ $=-\cos x$
5.2	$\dfrac{\cos(90°+\theta)}{\sin(\theta-180°)+3\sin(-\theta)}$ $=\dfrac{-\sin\theta}{\sin(-(180°-\theta))-3\sin\theta}$ $=\dfrac{-\sin\theta}{-\sin\theta-3\sin\theta}$ $=\dfrac{-\sin\theta}{-4\sin\theta}$ $=\dfrac{1}{4}$

5.3

$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$

$\cos x + 2\sin x = 0$ or $3\sin 2x - 1 = 0$

$\tan x = -\frac{1}{2}$ $\quad$ $\sin 2x = \frac{1}{3}$

ref $\angle = 26{,}565...°$ $\quad$ ref $\angle = 19{,}471...°$

$x = 153{,}43° + k.180° \; ; \; k \in Z$ $\quad$ $x = 9{,}74° + k.180°; \; k \in Z$

OR/*OF* $\quad$ or

$x = 153{,}43° + k.360° \; ; \; k \in Z$ $\quad$ $x = 80{,}26° + k.180°;$ $k \in Z$

or

$x = 333{,}43° + k.360° \; ; \; k \in Z$

5.4.1

$$\text{LHS} = \cos(x+y).\cos(x-y)$$
$$= [\cos x.\cos y - \sin x.\sin y][\cos x.\cos y + \sin x.\sin y]$$
$$= \cos^2 x.\cos^2 y - \sin^2 x.\sin^2 y$$
$$= (1-\sin^2 x)(1-\sin^2 y) - \sin^2 x.\sin^2 y$$
$$= 1 + \sin^2 x.\sin^2 y - \sin^2 x - \sin^2 y - \sin^2 x.\sin^2 y$$
$$= 1 - \sin^2 x - \sin^2 y \quad = \text{RHS}$$

5.4.2

$$1 - \sin^2 45° - \sin^2 15°$$
$$= \cos(45° + 15°).\cos(45° - 15°)$$
$$= \cos 60°.\cos 30°$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{4}$$

OR

$$
\begin{aligned}
&1-\sin^2 45^\circ-\sin^2 15^\circ \\
&=\sin^2 15^\circ+\cos^2 15^\circ-\sin^2 45^\circ-\sin^2 15^\circ \\
&=\cos^2 15^\circ-\left(\frac{\sqrt{2}}{2}\right)^2 \\
&=\cos^2 15^\circ-\frac{1}{2} \\
&=\frac{2\cos^2 15^\circ-1}{2} \\
&=\frac{\cos 30^\circ}{2} \\
&=\frac{\sqrt{3}}{2}\times\frac{1}{2} \\
&=\frac{\sqrt{3}}{4}
\end{aligned}
$$

OR

$$
\begin{aligned}
&1-\sin^2 45^\circ-\sin^2 15^\circ \\
&=\cos^2 45^\circ-\sin^2(45^\circ-30^\circ) \\
&=\left(\frac{1}{\sqrt{2}}\right)^2-(\sin 45^\circ\cos 30^\circ-\cos 45^\circ\sin 30^\circ)^2 \\
&=\frac{1}{2}-\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)^2 \\
&=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2 \\
&=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2 \\
&=\frac{1}{2}-\left(\frac{3}{8}-\frac{\sqrt{3}}{4}+\frac{1}{8}\right) \\
&=\frac{\sqrt{3}}{4}
\end{aligned}
$$

5.5.1	$16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 8\sin x.\cos x(2\cos^2 x - 1)$ $= 4\sin 2x(\cos 2x)$ $= 2\sin 4x$ **OR/*OF*** $16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 16\cos^2 x\left(\frac{1}{2}\sin 2x\right) - 8\left(\frac{1}{2}\sin 2x\right)$ $= 8\left(2\cos^2 x - 1\right)\left(\frac{1}{2}\sin 2x\right)$ $= 4\sin 2x.\cos 2x$ $= 2\sin 4x$
5.5.2	$16\sin x.\cos^3 x - 8\sin x.\cos x = 2\sin 4x$ Minimum at $x = 67{,}5°$

6.1	$180°$
6.2.1	$k=\sqrt{3}=1,73$
6.2.2	$B(-120°;\sqrt{3})$
6.3	Range of g: $y\in[-2;2]$ Range of $2g(x)$: $y\in[-4;4]$ **OR/*OF*** Range of g: $-2\le y\le 2$ Range of $2g(x)$: $-4\le y\le 4$
6.4	$x\in[-65°;-5°]$ **OR/*OF*** $-65°\le x\le -5°$
6.5	$\sin x.\cos x=p$ $4\sin x.\cos x=4p$ $2\sin 2x=4p$ $4p=\pm 2$ $\therefore\ p=-\frac{1}{2}$ or $\frac{1}{2}$

7.1

$$AD^2 = AB^2 + BD^2$$

$$AD^2 = \left(\sqrt{5}p\right)^2 + (2p)^2$$

$$AD^2 = 9p^2$$

$$AD = 3p$$

7.2

$$\frac{CD}{\sin(135° - x)} = \frac{3p}{\sin x}$$

$$CD = \frac{3p\sin(135° - x)}{\sin x}$$

$$CD = \frac{3p(\sin 135°\cos x - \cos 135°\sin x)}{\sin x}$$

$$CD = \frac{3p(\sin 45°\cos x + \cos 45°\sin x)}{\sin x}$$

$$CD = \frac{3p\left(\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x\right)}{\sin x}$$

$$CD = \frac{3p\left(\frac{\sqrt{2}}{2}\right)(\cos x + \sin x)}{\sin x}$$

$$CD = \frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$$

7.3

$$\text{Area } \Delta ADC = \frac{1}{2}(AD)(CD)\sin A\hat{D}C$$

$$= \frac{1}{2}(3p)\left(\frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}\right)(\sin 45°)$$

$$= \frac{1}{2}(30)\left(\frac{30(\sin 110° + \cos 110°)}{\sqrt{2}\sin 110°}\right)\sin 45°$$

$$= 143{,}11\,m^2$$

Paper B

5.1

$$\begin{aligned}
&\tan(-x).\cos x.\sin(x-180°)-1\\
&=-\tan x.\cos x.\sin(-(180°-x))-1\\
&=\frac{-\sin x}{\cos x}.\cos x.(-\sin x)-1\\
&=\sin^2 x-1\\
&=-\cos^2 x
\end{aligned}$$

5.2.1

$$\begin{aligned}
&\cos 215°\\
&=-\cos 35°\\
&=-m
\end{aligned}$$

5.2.2

$$\begin{aligned}
&\sin 20°\\
&=\cos 70°\\
&=\cos 2(35°)\\
&=2\cos^2 35°-1\\
&=2m^2-1
\end{aligned}$$

OR

$$\begin{aligned}
&=\sin(55°-35°)\\
&=\sin 55°\cos 35°-\cos 55°\sin 35°\\
&=m.m-\sqrt{1-m^2}.\sqrt{1-m^2}\\
&=m^2-(1-m^2)\\
&=2m^2-1
\end{aligned}$$

5.3

$$\cos 4x.\cos x + \sin 4x.\sin x = -0,7$$
$$\cos(4x - x) = -0,7$$
$$\text{ref} \angle = 45,57\ldots°$$

$$3x = 180° - 45,57...° + k.360° \text{ or } 3x = 180° + 45,57...° + k.360°$$
$$3x = 134,43° + k.360° \quad \text{or} \quad 3x = 225,57° + k.360°$$
$$x = 44,81° + k.120°;\ k \in Z \qquad x = 75,19° + k.120°;\ k \in Z$$

5.4

$$\text{RHS} = \cos^2 x - \sin^2 x$$

$$\text{LHS} = \frac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x}$$
$$= \frac{\sin 4x.\cos 2x - \cos 4x.\sin 2x}{\frac{\sin 2x}{\cos 2x}}$$
$$= \sin(4x - 2x)\left(\frac{\cos 2x}{\sin 2x}\right)$$
$$= \cos 2x$$
$$= \cos^2 x - \sin^2 x$$

$$\text{LHS} = \text{RHS}$$

6.1

$$1 - 2\sin^2 x = -\sin x$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$\sin x = -\frac{1}{2} \quad \text{or} \quad \sin x = 1$$
$$\text{ref} \angle = 30° \qquad \text{ref} \angle = 90°$$
$$x = 210° + k.360° \qquad x = 90° + k.360°$$
$$\text{or } x = 330° + k.360°$$

$$x = -150° \text{ or } x = -30° \text{ or } x = 90°$$

OR

$\cos 2x = -\sin x$
$\cos 2x = -\cos(90° - x)$

$2x = 180° - (90° - x) + k.360°$ or $2x = 180° + (90° - x) + k.360°$
$2x = 90° + x + k.360°$ or $2x = 270° - x + k.360°$
$x = 90° + k.360°$ $x = 90° + k.120°$

$x = -150°$ or $x = -30°$ or $x = 90°$

OR

$\cos 2x = -\sin x$
$\cos 2x = \cos(90° + x)$

$2x = 90° + x + k.360°$ or $2x = 360° - (90° + x) + k.360°$
$x = 90° + k.360°$ or $3x = 270° + k.360°$
$x = 90° + k.120°$

$x = -150°$ or $x = -30°$ or $x = 90°$

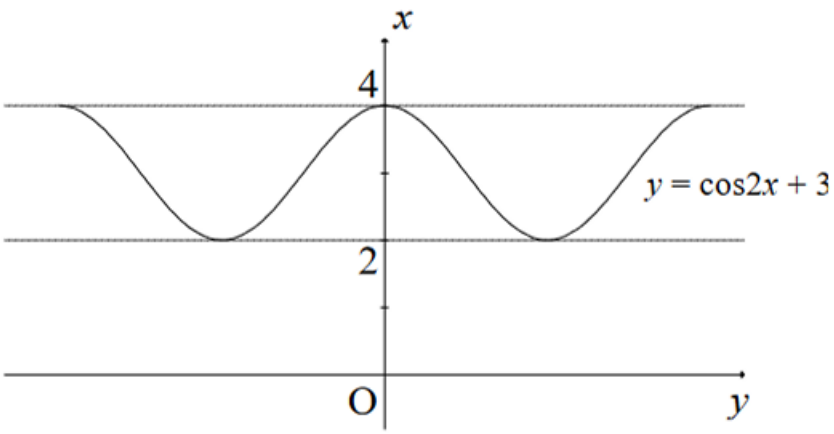
OR

$\cos 2x = -\sin x$
$\sin(90° - 2x) = -\sin x$

$90° - 2x = 180 + x + k.360°$ or $90° - 2x = 360° - x + k.360°$

$x = -30° + k.120°$ $x = -270° + k.360°$

$x = -150°$ or $x = -30°$ or $x = 90°$

6.2.1	A(–150°; 0,5) B(–30°; 0,5) AB = –30° – (–150°) AB = 120°
6.2.2	$x \in (0°; 90°)$ or $x \in (90°; 180°)$ **OR** $0° < x < 90°$ or $90° < x < 180°$
6.2.3	$\cos 2x = k - 3$ $k - 3 < -1$ or $k - 3 > 1$ $k < 2$ or $k > 4$ **OR**  $k < 2$ or $k > 4$

7.1

In ΔBCE:

$$\frac{CE}{\sin\hat{B}} = \frac{BC}{\sin B\hat{E}C}$$

$$\frac{CE}{\sin 30^\circ} = \frac{BC}{\sin 2x}$$

$$CE = \frac{BC\sin 30^\circ}{\sin 2x}$$

In ΔCDE:

$$\frac{DC}{CE} = \tan D\hat{E}C$$

$$DC = \frac{BC.\sin 30^\circ}{\sin 2x}(\tan x)$$

$$DC = \frac{BC}{4\sin x\cos x}\left(\frac{\sin x}{\cos x}\right)$$

$$DC = \frac{BC}{4\cos^2 x}$$

7.2

$$DC = \frac{BC}{4\cos^2 30^\circ}$$

$$= \frac{BC}{4\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{BC}{3}$$

$$\therefore BC = 3DC$$

But $AB = DC$ [opp sides of rectangle/*teenoorst. sye v reghoek*]

$$\therefore BC = 3AB$$

Area of rectangle $= (AB)(BC)$

$= (AB)(3AB)$

$= 3AB^2$

Paper C

5.1.1	$OP = \sqrt{(-7)^2 + (4)^2}$ $= \sqrt{65}$
5.1.2(a)	$\tan\theta = \frac{4}{-7}$
5.1.2(b)	$\cos(\theta - 180°) = -\cos\theta$ $= \frac{7}{\sqrt{65}}$
5.2	$\sin x\cos x + \sin x = 3\cos^2 x + 3\cos x$ $\sin x\cos x + \sin x - 3\cos^2 x - 3\cos x = 0$ $\sin x(\cos x + 1) - 3\cos x(\cos x + 1) = 0$ $(\cos x + 1)(\sin x - 3\cos x) = 0$ $\cos x = -1$ or $\sin x = 3\cos x$ $\tan x = 3$ $x = 180° + k.360°$ or $x = 71{,}57° + k.180°$; $k \in Z$ **OR/*OF*** $\sin x\cos x + \sin x = 3\cos^2 x + 3\cos x$ $\sin x\cos x + \sin x - 3\cos^2 x - 3\cos x = 0$ $\sin x(\cos x + 1) - 3\cos x(\cos x + 1) = 0$ $(\cos x + 1)(\sin x - 3\cos x) = 0$ $\cos x = -1$ or $\sin x = 3\cos x$ $\tan x = 3$ $x = 180° + k.360°$ or $x = 71{,}57° + k.360°$ or $x = 251{,}57° + k.360°$; $k \in Z$

5.3.1

$$\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{1+\cos 3x}{1+\cos 3x}$$

$$= \frac{(\sin 3x)(1+\cos 3x)}{(1-\cos 3x)(1+\cos 3x)}$$

$$= \frac{(\sin 3x)(1+\cos 3x)}{1-\cos^2 3x}$$

$$= \frac{(\sin 3x)(1+\cos 3x)}{\sin^2 3x}$$

$$= \frac{1+\cos 3x}{\sin 3x}$$

$$= \text{RHS}$$

OR/*OF*

$$\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{\sin 3x}{\sin 3x}$$

$$= \frac{\sin^2 3x}{\sin 3x(1-\cos 3x)}$$

$$= \frac{1-\cos^2 3x}{\sin 3x(1-\cos 3x)}$$

$$= \frac{(1-\cos 3x)(1+\cos 3x)}{\sin 3x(1-\cos 3x)}$$

$$= \frac{1+\cos 3x}{\sin 3x}$$

$$= \text{RHS}$$

5.3.2

undefined when $\sin 3x = 0$ and $1-\cos 3x = 0$

$3x = 0°$ or $3x = 180°$ and $3x = 0°$ or $3x = 360°$

$x = 0°$ or $x = 60°$

6.1

$$\frac{\sin 10^\circ}{\cos 440^\circ} + \tan(360^\circ - \theta).\sin 2\theta$$

$$= \frac{\cos 80^\circ}{\cos 80^\circ} - \tan\theta(2\sin\theta\cos\theta)$$

$$= 1 - \frac{\sin\theta}{\cos\theta}(2\sin\theta\cos\theta)$$

$$= 1 - 2\sin^2\theta$$

$$= \cos 2\theta$$

6.2.1

$$\sin(60^\circ + 2x) + \sin(60^\circ - 2x) = k\cos 2x$$

$$(\sin 60^\circ\cos 2x + \cos 60^\circ\sin 2x) + (\sin 60^\circ\cos 2x - \cos 60^\circ\sin 2x) = k\cos 2x$$

$$2\sin 60^\circ\cos 2x = k\cos 2x$$

$$2\left(\frac{\sqrt{3}}{2}\right)\cos 2x = k\cos 2x$$

$$\therefore k = \sqrt{3}$$

6.2.2

$$\tan 60^\circ[\sin(60^\circ + 2x) + \sin(60^\circ - 2x)]$$

$$= \tan 60^\circ[k\cos 2x]$$

$$= \sqrt{3}\left(\sqrt{3}\cos 2x\right)$$

$$= 3(2\cos^2 x - 1)$$

$$= 3\left(2\left(\sqrt{t}\right)^2 - 1\right)$$

$$= 6\left(\sqrt{t}\right)^2 - 3$$

$$= 6t - 3$$

7.1	$A\left(0;\frac{1}{2}\right)\quad B\left(0;-\frac{1}{2}\right)$ $AB=\frac{1}{2}-\left(-\frac{1}{2}\right)$ $=1$ unit
7.2	Range of $f:\ y\in\left[-\frac{1}{2};\frac{1}{2}\right]$ Range of $3f(x)+2:\ \ y\in\left[\frac{1}{2};3\frac{1}{2}\right]$ **OR/*OF*** $\frac{1}{2}\le y\le 3\frac{1}{2}$
7.3	$x=90°$
7.4.1	$x\in(30°;\ 90°)\cup(210°;240°]$ **OR/*OF*** $30°<x<90°$ or $210°<x\le 240°$
7.4.2	$x\in(-55°;\ 125°)$ **OR/*OF*** $-55°<x<125°$

8.1

$$\frac{0,5}{AB} = \sin 15°$$

$$AB = \frac{0,5}{\sin 15°}$$

$$AB = 1,93 \text{ m}$$

8.2

$$BE^2 = AB^2 + AE^2 - 2(AB)(AE)\cos B\hat{A}E$$

$$BE^2 = (1,93)^2 + (0,915)^2 - 2(1,93)(0,915)(\cos 120°)$$

$$BE = 2,52 \text{ m}$$

8.3

$$BF = FD = \frac{5}{7}(2,52) = 1,80 \text{ m}$$

$$\text{Area } \Delta BFD = \frac{1}{2}(BF)(FD)\sin B\hat{F}D$$

$$= \frac{1}{2}(1,8)(1,8)(\sin 75°)$$

$$= 1,56 \text{ m}^2$$

Paper D

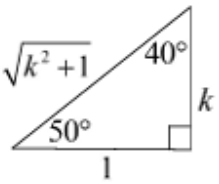
5.1.1

$r^2 = k^2 + 1^2$ [Pythagoras]

$r = \sqrt{k^2+1}$

$\cos 40^\circ$

$= \dfrac{k}{\sqrt{k^2+1}}$



5.1.2

$$\frac{2\sin 25^\circ \cos 25^\circ}{-2+4\sin^2 25^\circ}$$

$$= \frac{\sin 50^\circ}{-2\left(1-2\sin^2 25^\circ\right)}$$

$$= \frac{\sin 50^\circ}{-2\cos 50^\circ}$$

$$= \left(\frac{k}{\sqrt{k^2+1}}\right) \div \left(\frac{-2}{\sqrt{k^2+1}}\right)$$

OR $= -\dfrac{1}{2}\tan 50^\circ$

$$= -\frac{1}{2}k$$

5.1.3

$$\sin 10^\circ = \sin\left(50^\circ - 40^\circ\right)$$

$$= \sin 50^\circ \cos 40^\circ - \cos 50^\circ \sin 40^\circ$$

$$= \left(\frac{k}{\sqrt{k^2+1}}\right)\left(\frac{k}{\sqrt{k^2+1}}\right) - \left(\frac{1}{\sqrt{k^2+1}}\right)\left(\frac{1}{\sqrt{k^2+1}}\right)$$

$$= \frac{k^2-1}{k^2+1}$$

OR

$$\sin 10^\circ$$

$$= \cos 80^\circ$$

$$= \cos 2\left(40^\circ\right)$$

$$= 2\cos^2 40^\circ - 1$$

$$= 2\left(\frac{k}{\sqrt{k^2+1}}\right)^2 - 1$$

$$= \frac{2k^2}{k^2+1} - 1$$

$$= \frac{k^2-1}{k^2+1}$$

OR

$$\begin{aligned}\sin 10^\circ &= \sin(60^\circ - 50^\circ)\\ &= \sin 60^\circ \cos 50^\circ - \cos 60^\circ \sin 50^\circ\\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{k^2+1}}\right) - \left(\frac{1}{2}\right)\left(\frac{k}{\sqrt{k^2+1}}\right)\\ &= \frac{\sqrt{3}-k}{2\sqrt{k^2+1}}\end{aligned}$$

5.2.1

$$\begin{aligned}&\frac{\sin(540^\circ + x).\cos(90^\circ + x)}{\sin(-x)}\\ &= \frac{(-\sin x)(-\sin x)}{(-\sin x)}\\ &= -\sin x\end{aligned}$$

5.2.2

$x \in (180^\circ; 360^\circ)$

OR

$180^\circ < x < 360^\circ$

6.1

$$\begin{aligned}
\text{LHS} &= \left[\tan(180^\circ - x)\right](1-\cos^2 x)+\cos^2 x \\
&= (-\tan x)(\sin^2 x)+\cos^2 x \\
&= \left(-\frac{\sin x}{\cos x}\right)(\sin^2 x)+\cos^2 x \\
&= -\frac{\sin^3 x}{\cos x}+\cos^2 x \\
&= \frac{\sin^3 x-\cos^3 x}{-\cos x} \\
&= \frac{(\sin x-\cos x)(\sin^2 x+\sin x.\cos x+\cos^2 x)}{-\cos x} \\
&= \frac{(\sin x-\cos x)(1+\sin x.\cos x)}{-\cos x} \\
&= \text{RHS}
\end{aligned}$$

OR

$$\begin{aligned}
\text{RHS} &= \frac{(\sin x-\cos x)(1+\sin x.\cos x)}{-\cos x} \\
&= \frac{(\sin x-\cos x)(\cos^2 x+\sin^2 x+\sin x.\cos x)}{-\cos x} \\
&= \frac{\sin x\cos^2 x+\sin^3 x+\sin^2 x\cos x-\cos^3 x-\sin^2 x\cos x-\sin x.\cos^2 x}{-\cos x} \\
&= \frac{\sin^3 x-\cos^3 x}{-\cos x} \\
&= \frac{\sin^3 x}{-\cos x}+\cos^2 x \\
&= \frac{-\sin x}{\cos x}(\sin^2 x)+\cos^2 x \\
&= -\tan x(1-\cos^2 x)+\cos^2 x \\
&= \text{LHS}
\end{aligned}$$

6.2 $\sin^2 x;\ \cos^2 x;\ \frac{1}{2}\sin 2x$

$\cos^2 x-\sin^2 x=\frac{1}{2}\sin 2x-\cos^2 x$

$\cos^2 x-\sin^2 x=\frac{1}{2}(2\sin x\cos x)-\cos^2 x$

$\cos^2 x-\sin^2 x=\sin x\cos x-\cos^2 x$

$2\cos^2 x-\sin x\cos x-\sin^2 x=0$

$(2\cos x+\sin x)(\cos x-\sin x)=0$

$2\cos x=-\sin x$ or $\cos x=\sin x$

$\tan x=-2$ or $\tan x=1$

ref $\angle = 63,43^\circ$ or ref $\angle = 45^\circ$

$x=116,57^\circ+k.180^\circ$ or $x\neq 45^\circ+k.180^\circ;k\in Z$

OR

$\sin^2 x;\ \cos^2 x;\ \frac{1}{2}\sin 2x$

$\cos^2 x-\sin^2 x=\frac{1}{2}\sin 2x-\cos^2 x$

$\cos^2 x-\sin^2 x=\frac{1}{2}(2\sin x\cos x)-\cos^2 x$

$\cos^2 x-\sin^2 x=\sin x\cos x-\cos^2 x$

$\cos^2 x-\sin^2 x-\sin x\cos x+\cos^2 x=0$

$(\cos x-\sin x)(\cos x+\sin x)+\cos x(\cos x-\sin x)=0$

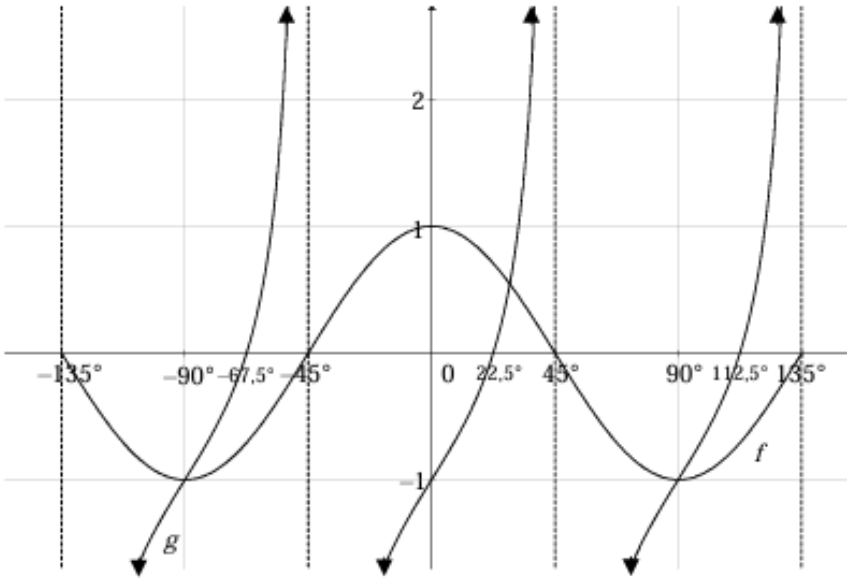
$(\cos x-\sin x)(\cos x+\sin x+\cos x)=0$

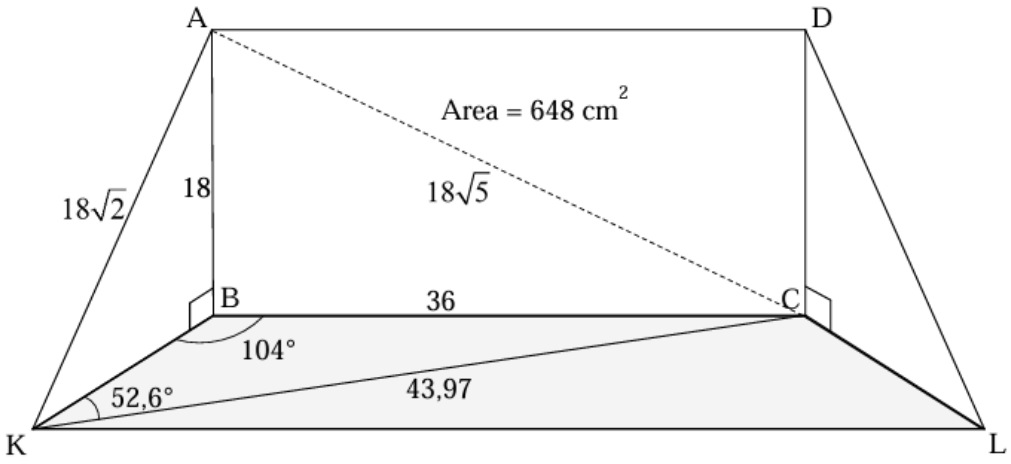
$\cos x=\sin x$ or $2\cos x=-\sin x$

$\tan x=1$ $\tan x=-2$

ref $\angle = 45^\circ$ ref $\angle = 63,43^\circ$

$x\neq 45^\circ+k.180^\circ;k\in Z$ $x=116,57^\circ+k.180^\circ$

7.1	$180°$
7.2	
7.3	$f(x)=\cos 2x$ $h(x)=\cos 2(x+45°)$ $=\cos(2x+90°)$ $=-\sin 2x$
7.4	$y\in[-1;1]$ **OR** $-1\le y\le 1$
7.5	$\tan 2x-1=0$ $\tan 2x=1$ $2x=45°$ $x=22,5°$ $(1-\tan 2x)(\cos 2x)\ge 0$ $-(\tan 2x-1)(\cos 2x)\ge 0$ $(\tan 2x-1)(\cos 2x)\le 0$ $x\in[0°;22,5°]\cup[112,5°;135°)$ **OR** $0°\le x\le 22,5°$ or $112,5°\le x<135°$



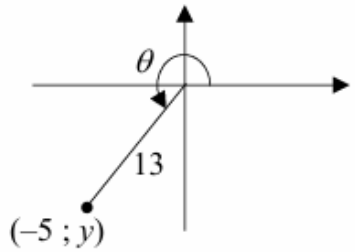
8.1	$\text{Area of ABCD} = BC \times AB$ $648 = 2AB \times AB$ $AB^2 = 324$ $AB = 18 \text{ cm}$
8.2	$AC^2 = AB^2 + BC^2 \quad [\text{Pythagoras}]$ $= 18^2 + 36^2$ $AC = \sqrt{1620} = 18\sqrt{5} = 40,25 \text{ cm}$
8.3	$\frac{KC}{\sin K\hat{B}C} = \frac{BC}{\sin B\hat{K}C}$ $\frac{KC}{\sin 104°} = \frac{36}{\sin 52,6°}$ $KC = \frac{36 \sin 104°}{\sin 52,6°}$ $KC = 43,97 \text{ cm}$
8.4	$AK^2 = AB^2 + BK^2 \quad [\text{Pythagoras}]$ $= 18^2 + 18^2$ $AK = \sqrt{648} = 18\sqrt{2} \text{ cm} = 25,46 \text{ cm}$ $KC^2 = AK^2 + AC^2 - 2AK.AC \cos K\hat{A}C$ $(43,97)^2 = (18\sqrt{2})^2 + (18\sqrt{5})^2 - 2(18\sqrt{2})(18\sqrt{5})(\cos K\hat{A}C)$ $\cos K\hat{A}C = 0,16...$ $K\hat{A}C = 80,60°$

Paper E

5.1.1

$y^2 = \sqrt{13^2 - (-5)^2}$ [Pythagoras]

$y = -12$



$\sin^2\theta$

$= \left(-\frac{12}{13}\right)^2$

$= \frac{144}{169}$

OR/*OF*

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = 1 - \left(-\frac{5}{13}\right)^2$

$\sin^2\theta = \frac{144}{169}$

5.1.2

$\tan(360° - \theta)$

$= -\tan\theta$

$= -\left(\frac{-12}{-5}\right)$

$= -\frac{12}{5}$

5.1.3

$\cos(\theta - 135°)$

$= \cos\theta\cos 135° + \sin\theta\sin 135°$

$= \cos\theta(-\cos 45°) + \sin\theta(\sin 45°)$

$= \left(-\frac{5}{13}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{12}{13}\right)\left(\frac{\sqrt{2}}{2}\right)$ OR $\left(-\frac{5}{13}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{12}{13}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= -\frac{7\sqrt{2}}{26}$ $= -\frac{7}{13\sqrt{2}}$

5.2

$$\frac{2\cos(180^\circ - x)\sin(-x)}{1-2\cos^2(90^\circ - x)}$$

$$= \frac{2(-\cos x)(-\sin x)}{1-2\sin^2 x}$$

$$= \frac{2\sin x\cos x}{\cos 2x}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

5.3

$$(\tan 92^\circ)(\tan 94^\circ)(\tan 96^\circ)...(\tan 176^\circ)(\tan 178^\circ)$$

$$= \left(\frac{\sin 92^\circ}{\cos 92^\circ}\right)\left(\frac{\sin 94^\circ}{\cos 94^\circ}\right)\left(\frac{\sin 96^\circ}{\cos 96^\circ}\right)...\left(\frac{\sin 176^\circ}{\cos 176^\circ}\right)\left(\frac{\sin 178^\circ}{\cos 178^\circ}\right)$$

$$= \left(\frac{\cos 2^\circ}{-\sin 2^\circ}\right)\left(\frac{\cos 4^\circ}{-\sin 4^\circ}\right)\left(\frac{\cos 6^\circ}{-\sin 6^\circ}\right)...\left(\frac{\sin 4^\circ}{-\cos 4^\circ}\right)\left(\frac{\sin 2^\circ}{-\cos 2^\circ}\right)$$

$$= 1$$

OR

$$(\tan 92^\circ)(\tan 94^\circ)(\tan 96^\circ)...(\tan 176^\circ)(\tan 178^\circ)$$

$$= (\tan 92^\circ)(\tan 94^\circ)(\tan 96^\circ)....(-\tan 4^\circ)(-\tan 2^\circ)$$

$$= \left(\frac{\sin 92^\circ}{\cos 92^\circ}\right)\left(\frac{\sin 94^\circ}{\cos 94^\circ}\right)\left(\frac{\sin 96^\circ}{\cos 96^\circ}\right)...\left(-\frac{\sin 4^\circ}{\cos 4^\circ}\right)\left(-\frac{\sin 2^\circ}{\cos 2^\circ}\right)$$

$$= \left(\frac{\cos 2^\circ}{-\sin 2^\circ}\right)\left(\frac{\cos 4^\circ}{-\sin 4^\circ}\right)\left(\frac{\cos 6^\circ}{-\sin 6^\circ}\right)...\left(-\frac{\sin 4^\circ}{\cos 4^\circ}\right)\left(-\frac{\sin 2^\circ}{\cos 2^\circ}\right)$$

$$= 1$$

6.1

$$\begin{aligned}\text{LHS} &= 2\cos^2(45^\circ + x)\\ &= 2\cos^2(45^\circ + x) + 1 - 1\\ &= \cos\left[2(45^\circ + x)\right] + 1\\ &= \cos(90^\circ + 2x) + 1\\ &= (-\sin 2x) + 1\\ &= 1 - \sin 2x\\ &= \text{RHS}\end{aligned}$$

OR/*OF*

$$\begin{aligned}\text{LHS} &= 2\cos^2(45^\circ + x)\\ &= 2\left(\cos(45^\circ + x)\right)^2\\ &= 2(\cos 45^\circ \cos x - \sin 45^\circ \sin x)^2\\ &= 2\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right)^2\\ &= 2\left(\frac{1}{2}\cos^2 x - \sin x \cos x + \frac{1}{2}\sin^2 x\right)\\ &= \cos^2 x - 2\sin x \cos x + \sin^2 x\\ &= 1 - \sin 2x\\ &= \text{RHS}\end{aligned}$$

6.2.1

$$\begin{aligned}\text{LHS} &= \sin(A - B) - \sin(A + B)\\ &= \sin A\cos B - \cos A\sin B - (\sin A\cos B + \cos A\sin B)\\ &= \sin A\cos B - \cos A\sin B - \sin A\cos B - \cos A\sin B\\ &= -2\cos A\sin B\\ &= \text{RHS}\end{aligned}$$

6.2.2

$$\begin{aligned}&\sin 4x - \sin 10x\\ &= \sin(7x - 3x) - \sin(7x + 3x)\\ &= -2\cos 7x\sin 3x\end{aligned}$$

6.2.3

$$\begin{aligned}&\sin 4x - \sin 10x = \sin 3x\\ &-2\cos 7x\sin 3x = \sin 3x\\ &2\cos 7x\sin 3x + \sin 3x = 0\\ &\sin 3x(2\cos 7x + 1) = 0\end{aligned}$$

$\sin 3x = 0$ or $\cos 7x = -\frac{1}{2}$

$3x = 0^\circ$ $\quad 7x = 120^\circ$ or $7x = 240^\circ$

$x = 0^\circ$ $\quad x = 17{,}14^\circ$ $\quad x = 34{,}29^\circ$

N/A

7.1	Range of f: $y \in [-1\,;\,3]$ **OR/*OF*** $-1 \le y \le 3$
7.2	Period of g: $180°$
7.3	f increasing: $x \in (-180°\,;\,0°)$ **OR/*OF*** $-180° < x < 0°$
7.4.1	$g(x).f'(x) < 0$ $x \in (-90°\,;\,0°) \cup (0°\,;\,90°)$ **OR/*OF*** $-90° < x < 0°$ or $0° < x < 90°$
7.4.2	$\cos x \le -\frac{1}{2}$ $2\cos x + 1 \le 0$ $x \in [-180°\,;\,-120°] \cup [120°\,;\,180°]$ **OR/*OF*** $2\cos x + 1 \le 0$ $-180° \le x \le -120°$ or $120° \le x \le 180°$
7.5	$g(x) = \sin 2x$ $h(x) = \sin 2(x - 45°)$ $= \sin(2x - 90°)$ $= -\sin(90° - 2x)$ $= -\cos 2x$

8.1	$\tan y = \frac{p}{\text{AB}}$ $\text{AB} = \frac{p}{\tan y}$
8.2	In ΔBAC: $\frac{\sin \text{B}\hat{\text{A}}\text{C}}{\text{BC}} = \frac{\sin \text{A}\hat{\text{C}}\text{B}}{\text{AB}}$ $\frac{\sin(180° - 2x)}{2p} = \frac{\sin x}{\left(\frac{p}{\tan y}\right)}$ $\frac{\sin 2x}{2p} = \sin x \times \left(\frac{\tan y}{p}\right)$ $\frac{2\sin x \cos x}{2p} = \sin x \times \left(\frac{\tan y}{p}\right)$ $2\cos x = \left(\frac{\tan y}{p}\right)(2p)$ $\cos x = \tan y$

8.2 **OR/*OF***

In ΔBAC:

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos B\hat{A}C$$

$$(2p)^2 = \left(\frac{p}{\tan y}\right)^2 + \left(\frac{p}{\tan y}\right)^2 - 2\left(\frac{p}{\tan y}\right)\left(\frac{p}{\tan y}\right)\cos(180° - 2x)$$

$$4p^2 = \frac{2p^2}{\tan^2 y} - \frac{2p^2(-\cos 2x)}{\tan^2 y}$$

$$4p^2\tan^2 y = 2p^2(1+\cos 2x)$$

$$\tan^2 y = \frac{1+2\cos^2 x - 1}{2}$$

$$\tan^2 y = \cos^2 x$$

$$\tan y = \cos x$$

8.3

$$\cos x = \tan y$$

$$\tan y = \cos 60°$$

$$\tan y = 0{,}5$$

$$y = 26{,}57°$$

Paper F

5.1.1

$r = 5$

$\cos A = -\frac{3}{5}$

5.1.2

$\cos 2A = 2\cos^2 A - 1$

$= 2\left(-\frac{3}{5}\right)^2 - 1$

$= -\frac{7}{25}$

OR/*OF*

$\cos 2A = \cos^2 A - \sin^2 A$

$= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$

$= -\frac{7}{25}$

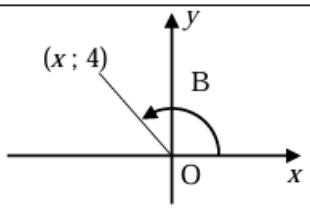
OR/*OF*

$\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(-\frac{4}{5}\right)^2$

$= -\frac{7}{25}$

5.1.3



$x = -3$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$= \left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{12}{25} + \frac{12}{25}$

$= \frac{24}{25}$

5.2

$$\frac{\cos\left(\frac{\alpha}{2}-45°\right)\sin\left(\frac{\alpha}{2}-45°\right)}{2}$$

$$=\frac{2\cos\left(\frac{\alpha}{2}-45°\right)\sin\left(\frac{\alpha}{2}-45°\right)}{2.2}$$

$$=\frac{\sin(\alpha-90°)}{4}$$

$$=\frac{-\cos\alpha}{4}$$

$$=\frac{-p}{4} \quad \textbf{OR/}\textit{OF} \quad =-\frac{1}{4}p$$

OR/*OF*

$$\frac{\cos\left(\frac{\alpha}{2}-45°\right)\sin\left(\frac{\alpha}{2}-45°\right)}{2}$$

$$=\frac{\left[\cos\frac{\alpha}{2}\cos 45°+\sin\frac{\alpha}{2}\sin 45°\right]\left[\sin\frac{\alpha}{2}\cos 45°-\cos\frac{\alpha}{2}\sin 45°\right]}{2}$$

$$=\frac{\left[\frac{\sqrt{2}}{2}\cos\frac{\alpha}{2}+\frac{\sqrt{2}}{2}\sin\frac{\alpha}{2}\right]\left[\frac{\sqrt{2}}{2}\sin\frac{\alpha}{2}-\frac{\sqrt{2}}{2}\cos\frac{\alpha}{2}\right]}{2}$$

$$=\frac{\frac{1}{2}\sin^2\frac{\alpha}{2}-\frac{1}{2}\cos^2\frac{\alpha}{2}}{2}$$

$$=\frac{-\frac{1}{2}\left(\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}\right)}{2}$$

$$=-\frac{\cos 2\left(\frac{\alpha}{2}\right)}{4}$$

$$=-\frac{\cos\alpha}{4}$$

$$=-\frac{1}{4}p$$

6.1.1

$$\begin{aligned}\cos(x+y) &= \cos(x-(-y)) \\ &= \cos x\cos(-y)+\sin x\sin(-y) \\ &= \cos x\cos y-\sin x\sin y\end{aligned}$$

6.1.2

$$\begin{aligned}\text{LHS} &= \frac{\cos(90^\circ-x)\cos y+\sin(-y)\cos(180^\circ+x)}{\cos x\cos(360^\circ+y)+\sin(360^\circ-x)\sin y} \\ &= \frac{(\sin x)\cos y+(-\sin y)(-\cos x)}{\cos x(\cos y)+(-\sin x)\sin y} \\ &= \frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y} \\ &= \frac{\sin(x+y)}{\cos(x+y)} \\ &= \tan(x+y) \\ &= \text{RHS}\end{aligned}$$

6.2

$$\sqrt{6\sin^2 x-11\cos(90^\circ+x)+7}=2$$

$$6\sin^2 x-11\cos(90^\circ+x)+7=4$$

$$6\sin^2 x-11(-\sin x)+7=4$$

$$6\sin^2 x+11\sin x+3=0$$

$$(3\sin x+1)(2\sin x+3)=0$$

$\sin x=-\frac{1}{3}$ **OR/*OF*** $\sin x=-\frac{3}{2}$

ref$\angle = 19{,}47^\circ$ no solution

$x = 199{,}47^\circ$ or $x = 340{,}53^\circ$

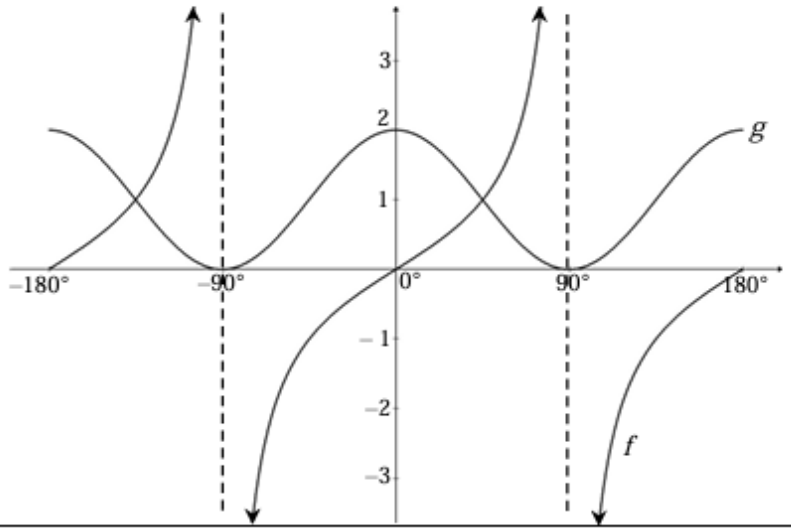
6.3.1

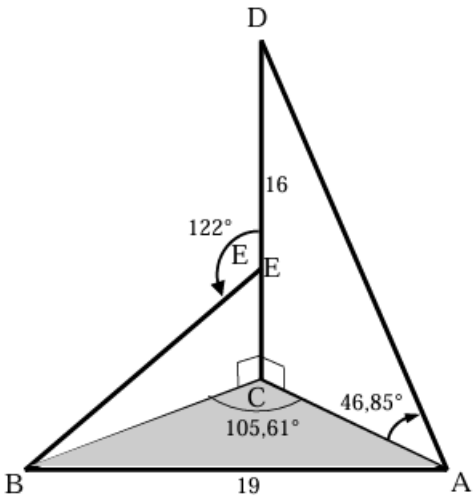
$$\begin{aligned}g(x) &= \frac{4-8\sin^2 x}{3} \\ &= \frac{4(1-2\sin^2 x)}{3} \\ &= \frac{4\cos 2x}{3}\end{aligned}$$

Maximum value of $\cos 2x$ is 1

$\therefore$ maximum value of $g(x)=\frac{4}{3}$

	OR/*OF*
	$4-8\sin^2 x$ is a maximum when $\sin^2 x$ is a minimum
	Minimum value of $\sin^2 x$ is 0
	$\therefore$ max. value of $g(x)=\dfrac{4-8(0)}{3}$
	$g(x)=\dfrac{4}{3}$
	OR/*OF*
	$\sin x=\dfrac{-(0)}{2\left(-\frac{8}{3}\right)}$
	$\sin x=0$
	$\therefore$ max. value of $g(x)=\dfrac{4-8(0)}{3}$
	$g(x)=\dfrac{4}{3}$
6.3.2	$x=180°$

7.1	$x = 90°$
7.2	$x = -180°$ or $x \in (-90° ; 0°]$ **OR/*OF*** $x = -180°$ or $-90° < x \leq 0°$
7.3.1	$180°$
7.3.2	
7.4	$2\cos^3 x - \sin x = 0$ $2\cos^3 x = \sin x$ $2\cos^2 x = \dfrac{\sin x}{\cos x}$ $2\cos^2 x = \tan x$ $2\cos^2 x - 1 = \tan x - 1$ $\cos 2x + 1 = \tan x$ $x = 45° + k.180°;\ k \in Z$ **OR/*OF*** $2\cos^3 x - \sin x = 0$ $\cos x\left(2\cos^2 x - \tan x\right) = 0$ $\cos x = 0$ or not valid or $2\cos^2 x = \tan x$ $2\cos^2 x - 1 + 1 = \tan x$ $\cos 2x + 1 = \tan x$ $x = 45° + k.180°;\ k \in Z$



8.1	$\tan \hat{DAC} = \frac{DC}{AC}$ $AC = \frac{16}{\tan\ 46{,}85^\circ}$ $AC = 15\ \text{m}$
8.2	$(AB)^2 = (BC)^2 + (AC)^2 - 2(BC)(AC)\cos \hat{BCA}$ $(19)^2 = x^2 + (15)^2 - 2x(15)\cos 105{,}61^\circ$ $x^2 + 8{,}07x - 136 = 0$ $x = \frac{-8{,}07 \pm \sqrt{(8{,}07)^2 - 4(1)(-136)}}{2(1)}$ $x = 8{,}30\ \text{m}$ or $x \neq -16{,}38\ \text{m}$ $\hat{BEC} = 58^\circ$ **OR/*OF*** $\hat{EBC} = 32^\circ$ $\tan \hat{BEC} = \frac{BC}{EC}$ $\tan \hat{EBC} = \frac{EC}{BC}$ $EC = \frac{8{,}3}{\tan 58^\circ}$ $EC = 8{,}3 \tan 32^\circ$ $EC = 5{,}19\ \text{m}$ $EC = 5{,}19\ \text{m}$ $DE = 10{,}81\ \text{m}$ $DE = 10{,}81\ \text{m}$

OR/*OF*

$$\frac{\sin 105{,}61^\circ}{19} = \frac{\sin C\hat{B}A}{15}$$

$$C\hat{B}A = 49{,}5^\circ$$

$$B\hat{A}C = 24{,}89^\circ$$

$$\frac{BC}{\sin 24{,}89^\circ} = \frac{19}{\sin 105{,}61^\circ}$$

$$BC = 8{,}3 \text{ m}$$

$B\hat{E}C = 58^\circ$	**OR/*OF***	$E\hat{B}C = 32^\circ$
$\tan B\hat{E}C = \frac{BC}{EC}$		$\tan E\hat{B}C = \frac{EC}{BC}$
$EC = \frac{8{,}3}{\tan 58^\circ}$		$EC = 8{,}3\tan 32^\circ$
$EC = 5{,}19$ m		$EC = 5{,}19$ m
$DE = 10{,}81$ m		$DE = 10{,}81$ m

Paper G

5.1	Period of $g = 360^\circ$
5.2	Amplitude of $f = \frac{1}{2}$
5.3	$f(180^\circ) - g(180^\circ)$ $= \frac{1}{2} - \left(-\frac{1}{2}\right)$ $= 1$
5.4.1	$x = 140{,}9^\circ$
5.4.2	$\sqrt{3}\sin x + \cos x \geq 1$ $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x \geq \frac{1}{2}$ $\sin x\cos 30^\circ + \cos x\sin 30^\circ \geq \frac{1}{2}$ $\sin(x + 30^\circ) \geq \frac{1}{2}$ $\sin(x + 30^\circ) = \frac{1}{2}$ at $x = 0^\circ$ or $x = 120^\circ$ $\therefore x \in [0^\circ; 120^\circ]$ **OR** $0^\circ \leq x \leq 120^\circ$

6.1.1 $\tan\theta = -\frac{12}{5}$ or $-2\frac{2}{5}$

6.1.2 $(OP)^2 = (-5)^2 + (12)^2$

$OP = 13$

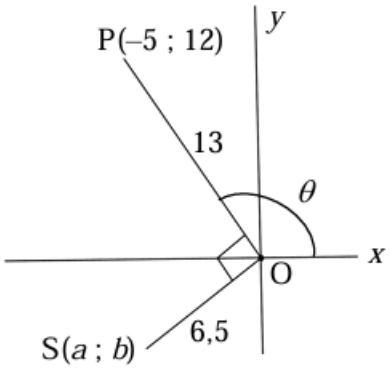
$\cos\theta = -\frac{5}{13}$

6.1.3 $\sin(\theta + 90°) = \frac{b}{6,5}$

$\cos\theta = \frac{b}{6,5}$

$\frac{-5}{13} = \frac{b}{6,5}$

$b = -\frac{5}{2}$



OR

$\cos(90° + \theta) = \frac{a}{6,5}$

$-\sin\theta = \frac{a}{6,5}$

$-\frac{12}{13} = \frac{a}{6,5} \quad \therefore a = -6$

$b = \sqrt{(6,5)^2 - (-6)^2} = -\frac{5}{2}$

6.2

$$\frac{\sin 2x.\cos(-x)+\cos 2x.\sin(360°-x)}{\sin(180°+x)}$$

$$=\frac{\sin 2x\cos x+\cos 2x(-\sin x)}{-\sin x}$$

$$=\frac{\sin(2x-x)}{-\sin x}$$

$$=\frac{\sin x}{-\sin x}$$

$$=-1$$

6.3

$$6\sin^2 x+7\cos x-3=0$$

$$6(1-\cos^2 x)+7\cos x-3=0$$

$$6-6\cos^2 x+7\cos x-3=0$$

$$6\cos^2 x-7\cos x-3=0$$

$$(3\cos x+1)(2\cos x-3)=0$$

$$\cos x=-\frac{1}{3} \quad \text{or} \quad \cos x=\frac{3}{2}\ (\text{N/A})$$

$$\therefore x=109{,}47°+k.360°;\ k\in Z \text{ or}$$

$$x=250{,}53°+k.360°;\ k\in Z$$

6.4

$$x+\frac{1}{x}=3\cos A$$

$$(3\cos A)^2=\left(x+\frac{1}{x}\right)^2$$

$$9\cos^2 A=x^2+\frac{1}{x^2}+2$$

$$9\cos^2 A=2+2$$

$$\cos^2 A=\frac{4}{9}$$

$$\cos 2A=2\cos^2 A-1$$

$$=2\left(\frac{4}{9}\right)-1$$

$$=-\frac{1}{9}$$

OR

$$x^2 - 2 + \frac{1}{x^2} = 0$$

$$\left(x - \frac{1}{x}\right)^2 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$3\cos A = 2 \quad \text{or} \quad 3\cos A = -2$$

$$\cos A = \frac{2}{3} \quad \text{or} \quad \cos A = -\frac{2}{3}$$

$$\cos 2A = 2\cos^2 A - 1$$

$$= 2\left(\pm\frac{2}{3}\right)^2 - 1$$

$$= -\frac{1}{9}$$

7.1

$$\tan 30° = \frac{\sqrt{3}\,r}{QS}$$

$$QS = \frac{\sqrt{3}r}{\tan 30°}$$

$$= \frac{\sqrt{3}r}{\frac{1}{\sqrt{3}}} \quad or \quad \frac{\sqrt{3}r}{\frac{\sqrt{3}}{3}}$$

$$= 3r$$

OR

$$\tan 60° = \frac{QS}{\sqrt{3}r}$$

$$\sqrt{3} = \frac{QS}{\sqrt{3}r}$$

$$QS = 3r$$

7.2

$$\text{Area of flower garden} = \pi(3r)^2 - \pi r^2$$

$$= 9\pi r^2 - \pi r^2$$

$$= 8\pi r^2$$

7.3

$$RS^2 = r^2 + (3r)^2 - 2(r)(3r)\cos 2x$$

$$= r^2 + 9r^2 - 6r^2\cos 2x$$

$$= 10r^2 - 6r^2\cos 2x$$

$$= r^2(10 - 6\cos 2x)$$

$$RS = r\sqrt{10 - 6\cos 2x}$$

7.4

$$RS = 10\sqrt{10 - 6\cos 2(56)}$$

$$= 34{,}9966...$$

$$\approx 35 \text{ m}$$

Paper H

5.1

$$\frac{\sin 140^\circ . \sin(360^\circ - x)}{\cos 50^\circ . \tan(-x)}$$

$$= \frac{\sin 40^\circ (-\sin x)}{\sin 40^\circ (-\tan x)}$$

$$= \frac{-\sin x}{-\frac{\sin x}{\cos x}}$$

$$= \cos x$$

5.2

$$\text{LHS} = \frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540^\circ - x)} \qquad \text{RHS} = 2\cos x - 1$$

$$\text{LHS} = \frac{-2(1 - \cos^2 x) + \cos x + 1}{1 - (-\cos x)}$$

$$\text{LHS} = \frac{-2 + 2\cos^2 x + \cos x + 1}{1 + \cos x}$$

$$\text{LHS} = \frac{2\cos^2 x + \cos x - 1}{1 + \cos x}$$

$$\text{LHS} = \frac{(2\cos x - 1)(\cos x + 1)}{1 + \cos x}$$

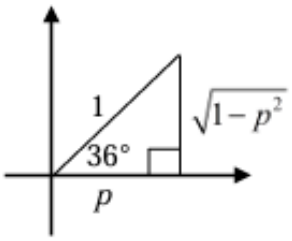
$$\text{LHS} = 2\cos x - 1$$

$$\therefore \text{LHS} = \text{RHS}$$

5.3.1

$$\sin 36^\circ = \sqrt{1 - p^2}$$

$$\tan 36^\circ = \frac{\sqrt{1 - p^2}}{p}$$



OR/*OF*

$$\cos^2 36^\circ = 1 - \sin^2 36^\circ$$

$$\cos 36^\circ = \sqrt{1 - (1 - p^2)}$$

$$= p$$

$$\tan 36^\circ = \frac{\sin 36^\circ}{\cos 36^\circ}$$

$$= \frac{\sqrt{1 - p^2}}{p}$$

5.3.2

$$\cos 108^\circ$$
$$= -\cos 72^\circ$$
$$= -\cos\ (2\times 36^\circ)$$
$$= -(2\cos^2 36^\circ - 1)$$
$$= -2p^2 + 1$$

OR/*OF*

$$\cos 108^\circ$$
$$= -\cos 72^\circ$$
$$= -\cos\ (2\times 36^\circ)$$
$$= -(1 - 2\sin^2 36^\circ)$$
$$= -1 + 2\left(\sqrt{1-p^2}\right)^2$$
$$= -1 + 2(1-p^2)$$
$$= -2p^2 + 1$$

OR/*OF*

$$\cos 108^\circ$$
$$= -\cos 72^\circ$$
$$= -\cos\ (2\times 36^\circ)$$
$$= -(\cos^2 36^\circ - \sin^2 36^\circ)$$
$$= -\left(p^2 - \left(\sqrt{1-p^2}\right)^2\right)$$
$$= -\left(p^2 - (1-p^2)\right)$$
$$= -2p^2 + 1$$

OR/*OF*

$$\cos 108^\circ$$
$$= \cos(2\times 54^\circ)$$
$$= 2\cos^2 54^\circ - 1$$
$$= 2(1-p^2) - 1$$
$$= 1 - 2p^2$$

OR/*OF*

$$\cos 108^\circ = \cos(72^\circ + 36^\circ)$$
$$= \cos 72^\circ \cos 36^\circ - \sin 72^\circ \sin 36^\circ$$
$$= (2\cos^2 36^\circ - 1)\cos 36^\circ - (2\sin 36^\circ \cos 36^\circ)\sin 36^\circ$$
$$= 2\cos^3 36^\circ - \cos 36^\circ - 2\cos 36^\circ \sin^2 36^\circ$$
$$= 2p^3 - p - 2p\left(\sqrt{1-p^2}\right)^2$$
$$= 2p^3 - p - 2p + 2p^3$$
$$= 4p^3 - 3p$$

6.1.1

$$\cos(\alpha+\beta)$$
$$=\cos(\alpha-(-\beta))$$
$$=\cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)$$
$$=\cos\alpha\cos\beta+\sin\alpha(-\sin\beta)$$
$$=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

6.1.2

$$2\cos 6x\cos 4x-\cos 10x+2\sin^2 x$$
$$=2\cos 6x\cos 4x-\cos(6x+4x)+2\sin^2 x$$
$$=2\cos 6x\cos 4x-(\cos 6x\cos 4x-\sin 6x\sin 4x)+2\sin^2 x$$
$$=\cos 6x\cos 4x+\sin 6x\sin 4x+2\sin^2 x$$
$$=\cos 2x+2\sin^2 x$$
$$=1-2\sin^2 x+2\sin^2 x$$
$$=1$$

6.2

$$\tan x=2\sin 2x$$
$$\frac{\sin x}{\cos x}=2(2\sin x\cos x)$$
$$\sin x=4\sin x\cos^2 x$$
$$4\sin x\cos^2 x-\sin x=0$$
$$\sin x(4\cos^2 x-1)=0$$

$$\sin x=0 \quad \text{or} \quad \cos^2 x=\frac{1}{4}$$
$$\cos x=-\frac{1}{2}$$

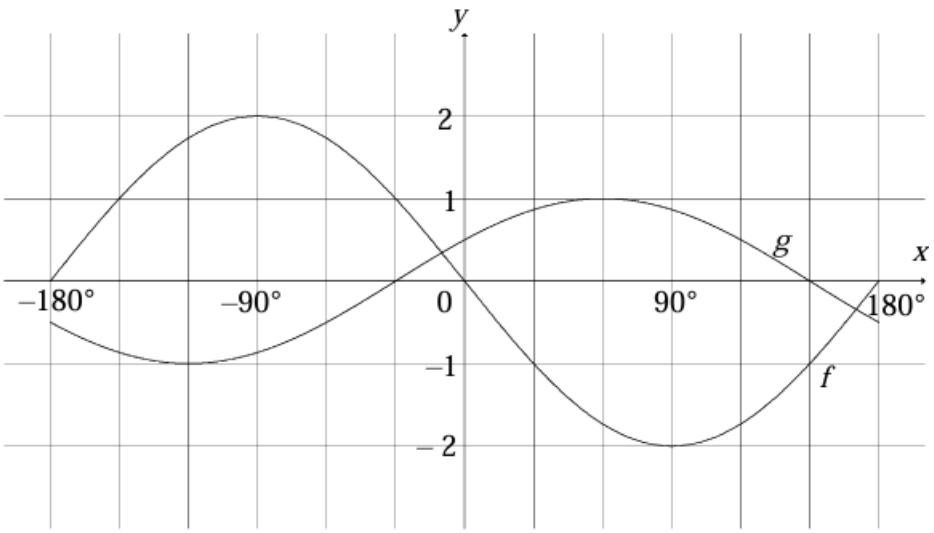
$$x=180°+k.360°;\ k\in Z \quad \text{or} \quad x=120°+k.360°;\ k\in Z$$
$$x=240°+k.360°;\ k\in Z$$

OR/*OF*

$$\tan x=2\sin 2x$$
$$\frac{\sin x}{\cos x}=4\sin x\cos x$$
$$\sin x=4\sin x\cos^2 x$$
$$4\sin x\cos^2 x-\sin x=0$$
$$4\sin x(1-\sin^2 x)-\sin x=0$$
$$3\sin x-4\sin^3 x=0$$
$$\sin x(3-4\sin^2 x)=0$$

$$\sin x=0 \quad \text{or} \quad \sin^2 x=\frac{3}{4}$$
$$\sin x=\frac{\sqrt{3}}{2} \quad \text{or} \quad \sin x=-\frac{\sqrt{3}}{2}$$

$$x=180^\circ+k.360^\circ, k\in Z \quad \text{or} \quad x=120^\circ+k.360^\circ, k\in Z$$
$$\text{or } x=240^\circ+k.360^\circ, k\in Z$$

7.1	
7.2	Period = 120°
7.3	$x = -30°$
7.4	Range of/*waardeversameling van g*: $y \in [-1;1]$ Range of/*Waardeversameling van* $\frac{1}{2}g$: $y \in \left[-\frac{1}{2};\frac{1}{2}\right]$ Range of/*Waardeversameling van* $\frac{1}{2}g+1$: $y \in \left[\frac{1}{2};\frac{3}{2}\right]$ **OR/*OF*** Range of/*Waardeversameling van* $\frac{1}{2}g+1$: $\frac{1}{2} \le y \le \frac{3}{2}$

8.1 In ΔSQR:

$$\frac{QS}{\sin x} = \frac{QR}{\sin(90° + x)}$$

$$\frac{QS}{\sin x} = \frac{5}{\cos x}$$

$$QS = \frac{5\sin x}{\cos x}$$

$$QS = 5\tan x$$

8.2

$$\frac{QT}{\sin(180° - 2x)} = \frac{TS}{\sin x}$$

$$\frac{QT}{\sin 2x} = \frac{5\tan x}{\sin x}$$

$$QT = \frac{5\tan x \sin 2x}{\sin x}$$

$$QT = \frac{5\left(\frac{\sin x}{\cos x}\right)(2\sin x\cos x)}{\sin x}$$

$$QT = \frac{5\sin x\,(2\sin x)}{\sin x}$$

$$QT = 10\sin x$$

OR/*OF*

$$QT^2 = QS^2 + TS^2 - 2QS.TS\cos Q\hat{S}T$$
$$QT^2 = (5\tan x)^2 + (5\tan x)^2 - 2(5\tan x).(5\tan x)\cos(180° - 2x)$$
$$QT^2 = 50\tan^2 x - 50\tan^2 x(-\cos 2x)$$
$$QT^2 = 50\tan^2 x(1+\cos 2x)$$
$$QT^2 = 50\tan^2 x(1+2\cos^2 x-1)$$
$$QT^2 = 50\tan^2 x(2\cos^2 x)$$
$$QT^2 = 100\frac{\sin^2 x}{\cos^2 x}(\cos^2 x)$$
$$QT^2 = 100\sin^2 x$$
$$QT = 10\sin x$$

OR/*OF*

$$TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$$
$$(5\tan x)^2 = (5\tan x)^2 + TQ^2 - 2(5\tan x).TQ.\cos x$$
$$0 = TQ^2 - 2(5\tan x).TQ.\cos x$$
$$0 = TQ\left[TQ - 10\tan x.\cos x\right]$$
$$TQ = 10\tan x.\cos x \quad (TQ \neq 0)$$
$$= 10\frac{\sin x}{\cos x}.\cos x$$
$$= 10\sin x$$

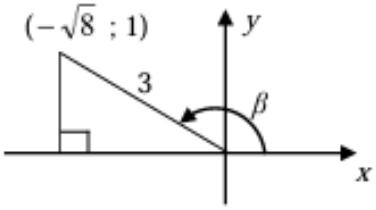
8.3

$$\text{Area of } \Delta TQR = \frac{1}{2}.TQ.QR\sin T\hat{Q}R$$
$$= \frac{1}{2}(10\sin 25°)(5)(\sin 70°)$$
$$= 9{,}93\,\text{unit}^2$$

Paper I

5.1.1

$\sin\beta = \frac{1}{3}$ $\qquad \beta \in (90°;270°)$



$x = -\sqrt{8} = -2\sqrt{2}$

$\cos\beta$

$= \frac{-2\sqrt{2}}{3}$

OR

$\sin\beta = \frac{1}{3}$ $\qquad \beta \in (90°;270°)$

$\cos^2\beta = 1 - \sin^2\beta$

$\cos^2\beta = 1 - \left(\frac{1}{3}\right)^2$

$\cos^2\beta = \frac{8}{9}$

$\cos\beta = \frac{-\sqrt{8}}{3}$

$= \frac{-2\sqrt{2}}{3}$

5.1.2

$\sin 2\beta$

$= 2\sin\beta\cos\beta$

$= 2\left(\frac{1}{3}\right)\left(\frac{-\sqrt{8}}{3}\right)$

$= \frac{-2\sqrt{8}}{9}$ **OR** $2\left(\frac{-2\sqrt{2}}{9}\right)$

$= \frac{-4\sqrt{2}}{9}$

5.1.3

$\cos(450° - \beta)$

$= \cos(90° - \beta)$

$= \sin\beta$

$= \frac{1}{3}$

OR

$$\cos(450^\circ - \beta)$$
$$= \cos 450^\circ \cos\beta + \sin 450^\circ \sin\beta$$
$$= \cos 90^\circ \cos\beta + \sin 90^\circ \sin\beta$$
$$= \sin\beta$$
$$= \frac{1}{3}$$

5.2.1

$$\text{LHS} = \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$$
$$= \frac{\cos^2 x\left(\cos^2 x + \sin^2 x\right)}{1+\sin x}$$
$$= \frac{1-\sin^2 x}{1+\sin x}$$
$$= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$$
$$= 1-\sin x$$
$$= \text{RHS}$$

OR

$$\text{LHS} = \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$$
$$= \frac{\cos^4 x + \left(1-\cos^2 x\right)\cos^2 x}{1+\sin x}$$
$$= \frac{\cos^4 x + \cos^2 x - \cos^4 x}{1+\sin x}$$
$$= \frac{1-\sin^2 x}{1+\sin x}$$
$$= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$$
$$= 1-\sin x$$
$$= \text{RHS}$$

OR

$$\text{RHS} = 1-\sin x$$
$$= (1-\sin x)\times\frac{1+\sin x}{1+\sin x}$$
$$= \frac{1-\sin^2 x}{1+\sin x}$$
$$= \frac{\cos^2 x}{1+\sin x}$$
$$= \frac{\cos^2 x\left(\sin^2 x + \cos^2 x\right)}{1+\sin x}$$
$$= \frac{\cos^4 x + \cos^2 x.\sin^2 x}{1+\sin x}$$
$$= \text{LHS}$$

5.2.2

$\sin x+1=0$

$\sin x=-1$

ref. $\angle = 90°$

$x = 270°$

5.2.3

$$y=\frac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$$

$y=1-\sin x$

$\therefore$ Minimum = 0

5.3.1

$\sin (A - B)$

$=\cos\left[90°-(A-B)\right]$

$=\cos\left[(90°-A)-(-B)\right]$

$=\cos(90°-A)\cos(-B)+\sin(90°-A)\sin(-B)$

$=\sin A\cos B+\cos A(-\sin B)$

$=\sin A\cos B-\cos A\sin B$

OR

$\sin (A - B)$

$=\cos\left[90°-(A-B)\right]$

$=\cos\left[(90°+B)-A\right]$

$=\cos(90°+B)\cos A+\sin(90°+B)\sin A$

$=-\sin B\cos A+\cos B\sin A$

$=\sin A\cos B-\cos A\sin B$

5.3.2

$\sin 48°\cos x-\cos 48°\sin x=\cos 2x$

$\sin(48°-x)=\cos 2x$

$\sin(48°-x)=\sin(90°-2x)$

$48°-x=90°-2x+k.360°$ or

$48°-x=180°-(90°-2x)+k.360°$

$x=42°+k.360°$ $\qquad -3x=42°+k.360°$

$x=-14°-k.120°$; $k\in Z$

OR

$\sin 48°\cos x-\cos 48°\sin x=\cos 2x$

$\sin(48°-x)=\cos 2x$

$\cos(90°-48°+x)=\cos 2x$

$\cos(42°+x)=\cos 2x$

$42°+x=2x+k.360°$ or $42°+x=360°-2x+k.360°$

$-x=-42°+k.360°$ $\qquad 3x=318°+k.360°$

$x=42°-k.360°$ $\qquad x=106°+k.120°$; $k\in Z$

5.4

$$\frac{\sin 3x+\sin x}{\cos 2x+1}$$
$$=\frac{\sin(2x+x)+\sin(2x-x)}{\cos 2x+1}$$
$$=\frac{\sin 2x\cos x+\cos 2x\sin x+\sin 2x\cos x-\cos 2x\sin x}{2\cos^2 x-1+1}$$
$$=\frac{2\sin 2x\cos x}{2\cos^2 x}$$
$$=\frac{2(2\sin x\cos x)\cos x}{2\cos^2 x}$$
$$=\frac{4\sin x\cos^2 x}{2\cos^2 x}$$
$$=2\sin x$$

OR

$$\frac{\sin 3x+\sin x}{\cos 2x+1}$$
$$=\frac{\sin(2x+x)+\sin x}{2\cos^2 x-1+1}$$
$$=\frac{\sin 2x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$$
$$=\frac{2\sin x\cos x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$$
$$=\frac{\sin x(2\cos^2 x+\cos 2x+1)}{2\cos^2 x}$$
$$=\frac{\sin x(2\cos^2 x+2\cos^2 x-1+1)}{2\cos^2 x}$$
$$=2\sin x$$

6.1	Period = $180°$
6.2	$y \in \left[-\frac{\sqrt{2}}{2}; 1\right]$ **OR** $y \in [-0,71; 1]$ **OR** $-\frac{\sqrt{2}}{2} \leq y \leq 1$
6.3.1	$x \in (45° ; 90°)$ **OR** $45° < x < 90°$
6.3.2	$f(x)+1 \leq 0$ $f(x) \leq -1$ $x \in [105° ; 165°]$ **OR** $105° \leq x \leq 165°$
6.4	$p(x) = -2\sin 2x$ $-2\sin 2x = -1$ **OR** $2\sin 2x = 1$ $k = 15°$ or $k = 75°$
6.5	$g(x) = -\cos(x+45°)$ $h(x) = -\cos(x+90°)$ $h(x) = \sin x$

7.1	$\text{Area } \Delta STK = \frac{1}{2}p(SK)\sin\alpha$ $q = \frac{1}{2}p(SK)\sin\alpha$ $SK = \frac{q}{\frac{1}{2}p\sin\alpha}$ $= \frac{2q}{p\sin\alpha}$
7.2	$R\hat{K}S = \beta$ $\frac{RS}{SK} = \tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$ **OR** $\frac{RS}{\sin\beta} = \frac{SK}{\sin(90° - \beta)}$ $RS\cos\beta = SK\sin\beta$ $RS = SK\tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$
7.3	$70 = \frac{2(2500)\tan 42°}{80\sin\alpha}$ $\sin\alpha = \frac{25}{28}\tan 42°$ **OR** $\sin\alpha = 0,80...$ $\alpha = 53,51°$

Paper J

5.1.1	$\sin 220°$ $=-\sin 40°$ $=-p$
5.1.2	$\cos^2 50°$ $=\sin^2 40°$ $=p^2$
5.1.3	$\cos(-80°)$ $=\cos 80°$ $=1-2\sin^2 40°$ $=1-2p^2$ **OR** $\cos(-80°)$ $=\cos 80°$ $=\cos(30°+50°)$ $=\cos 30°\cos 50°-\sin 30°\sin 50°$ $=\frac{\sqrt{3}p}{2}-\frac{\sqrt{1-p^2}}{2}$
5.2.1	$\text{LHS}=\tan x(1-\cos^2 x)+\cos^2 x$ $=\frac{\sin x}{\cos x}(\sin^2 x)+\cos^2 x$ $=\frac{\sin^3 x+\cos^3 x}{\cos x}$ $=\frac{(\sin x+\cos x)(\sin^2 x-\sin x\cos x+\cos^2 x)}{\cos x}$ $=\frac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$ $=\text{RHS}$

OR

$$\text{RHS} = \frac{(\sin x + \cos x)(1 - \sin x \cos x)}{\cos x}$$
$$= \frac{\sin x - \sin^2 x \cos x + \cos x - \sin x \cos^2 x}{\cos x}$$
$$= \tan x - \sin^2 x + 1 - \sin x \cos x$$
$$= \tan x + \cos^2 x - \sin x \cos x$$
$$= \tan x\left(1 - \frac{\sin x \cos x}{\tan x}\right) + \cos^2 x$$
$$= \tan x\left(1 - \frac{\sin x \cos x}{\frac{\sin x}{\cos x}}\right) + \cos^2 x$$
$$= \tan x\left(1 - \cos^2 x\right) + \cos^2 x$$
$$= \text{LHS}$$

5.2.2

$\cos x = 0$ or where $\tan x$ is undefined

$x = 90° + k.360°$ or $x = 270° + k.360°$

$x = 90°$ or $x = -90°$

5.3.1

$$\frac{\sin 150° + \cos^2 x - 1}{2}$$
$$= \frac{\sin 30° + \cos^2 x - 1}{2}$$
$$= \frac{\frac{1}{2} - \left(1 - \cos^2 x\right)}{2}$$
$$= \left(\frac{1}{2} - \sin^2 x\right) \times \frac{1}{2}$$
$$= \frac{1 - 2\sin^2 x}{4}$$
$$= \frac{\cos 2x}{4}$$

5.3.2

$$\frac{\sin 150° + \cos^2 x - 1}{2} = \frac{1}{25}$$
$$\frac{\cos 2x}{4} = \frac{1}{25}$$
$$\cos 2x = \frac{4}{25}$$
$$\text{ref}\angle = 80,79...°$$

$2x = 80,79...° + k.360°$ or $2x = 279,20...° + k.360°$

$x = 40,40° + k.180°$ or $x = 139,60° + k.180°$; $k \in Z$

OR

$$\frac{\sin 150° + \cos^2 x - 1}{2} = \frac{1}{25}$$

$$\sin 150° + \cos^2 x - 1 = \frac{2}{25}$$

$$\sin 30° + \cos^2 x - 1 = \frac{2}{25}$$

$$\cos^2 x = \frac{29}{50}$$

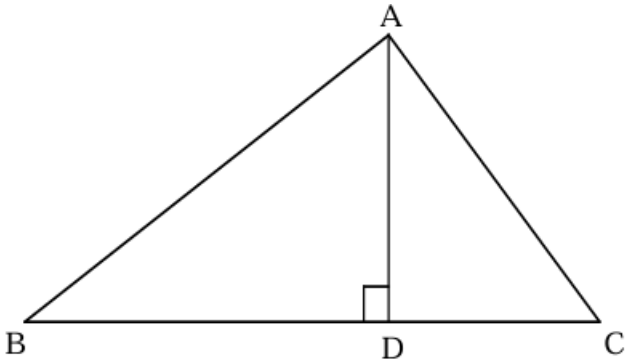
$$\cos x = \pm\sqrt{\frac{29}{50}}$$

$x = 40,40° + k.360°$ or $x = 319,60° + k.360°$; $k \in Z$

or

$x = 139,60° + k.360°$ or $x = 220,40° + k.360°$; $k \in Z$

6.1	Period = $360°$
6.2	Amplitude $= 1$
6.3	$a = -45°$
6.4	$\sin 2x = k$ $k = \sin(2\times 165°)$ **OR** $k = \sin\left(2\times(-75°)\right)$ $k = \sin 330°$ $\quad$ $k = \sin(-150°)$ $k = -\sin 30°$ $k = -\frac{1}{2}$ **OR** $k = \cos(165° - 45°)$ **OR** $k = \cos(-75° - 45°)$ $k = \cos 120°$ $\quad$ $k = \cos(-120°)$ $k = -\cos 60°$ $k = -\frac{1}{2}$
6.5	Points of intersection are translated $60°$ to the left $x = -15°$
6.6	$\sqrt{2}\sin 2x = \sin x + \cos x$ $\sin 2x = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$ $\sin 2x = \sin 45°\sin x + \cos 45°\cos x$ $\sin 2x = \cos(45° - x)$ **OR** $\sin 2x = \cos(x - 45°)$ $\therefore$ 2 roots in the interval $x \in [-90°; 90°]$



7.1.1	$\sin \hat{B} = \frac{AD}{AB}$ $AD = AB\sin \hat{B}$
7.1.2	$\text{Area of } \Delta ABC = \frac{1}{2}(BC)(AD)$ $\therefore \text{Area of } \Delta ABC = \frac{1}{2}(BC)(AB)\sin \hat{B}$

7.2.1 In ΔADB

$$\sin \alpha = \frac{AB}{AD}$$

$$AD = \frac{AB}{\sin \alpha}$$

In ΔABC

$$\sin \alpha = \frac{AB}{AC}$$

$$AC = \frac{AB}{\sin \alpha}$$

$$AD = AC$$

OR

In ΔADB and ΔACB

$AB = AB$	[common side]
$A\hat{B}D = A\hat{B}C = 90°$	[given]
$BD = BC$	[given]
$\Delta ADB \equiv \Delta ACB$	[S∠S]
$\therefore\ AD = AC$	

OR

In ΔADB and ΔACB

$A\hat{D}B = A\hat{C}B = \alpha$	[given]
$A\hat{B}D = A\hat{B}C = 90°$	[given]
$AB = AB$ **OR** $BD = BC$	[common side **OR** given]
$\therefore\ \Delta ADB \equiv \Delta ACB$	[∠∠S]
$\therefore AD = AC$	

OR

$AD^2 = AB^2 + DB^2$	[Pythagoras]
$AC^2 = AB^2 + BC^2$	[Pythagoras]
But $DB = BC$	[given]

$\therefore AD^2 = AC^2$

$\therefore AD = AC$

7.2.2

$$\frac{BD}{\sin\theta} = \frac{k}{\sin(180° - 2\theta)}$$

$$BD = \frac{k\sin\theta}{\sin 2\theta}$$

$$BD = \frac{k\sin\theta}{2\sin\theta\cos\theta}$$

$$BD = \frac{k}{2\cos\theta}$$

OR

$$BC^2 = k^2 + BD^2 - 2k(BD)\cos\theta$$

$$BD^2 = k^2 + BD^2 - 2k(BD)\cos\theta$$

$$k^2 - 2k(BD)\cos\theta = 0$$

$$2k(BD)\cos\theta = k^2$$

$$\therefore BD = \frac{k}{2\cos\theta}$$

7.2.3

$$\text{Area of } \Delta BCD = \frac{1}{2}(DC)(BD)\left(\sin C\hat{D}B\right)$$

$$= \frac{1}{2}k\left(\frac{k}{2\cos\theta}\right)\sin\theta$$

$$= \frac{1}{4}k^2\tan\theta$$

OR

$$\text{Area of } \Delta BCD = \frac{1}{2}(BD)(BC)\left(\sin(180° - 2\theta)\right)$$

$$= \frac{1}{2}\left(\frac{k}{2\cos\theta}\right)\left(\frac{k}{2\cos\theta}\right)(\sin 2\theta)$$

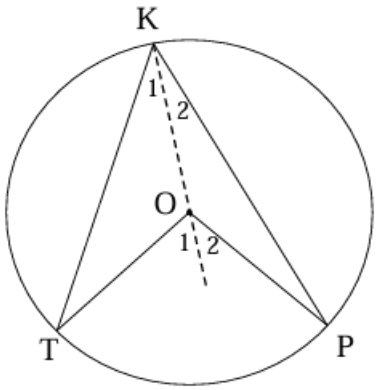
$$= \frac{2k^2\sin\theta\cos\theta}{8\cos\theta\cos\theta}$$

$$= \frac{1}{4}k^2\tan\theta$$

EUCLIDEAN GEOMETRY

Provide reason for ALL your statements in this section.

Paper A



9.1 Construction: Draw KO produced

$\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext $\angle$ of Δ/*buite* $\angle$ *van* Δ]

But $\hat{K}_1 = \hat{T}$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*]

$\therefore \hat{O}_1 = 2\hat{K}_1$

$\hat{O}_2 = \hat{K}_2 + P$ [ext $\angle$ of Δ/*buite* $\angle$ *van* Δ]

But $\hat{K}_2 = P$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*]

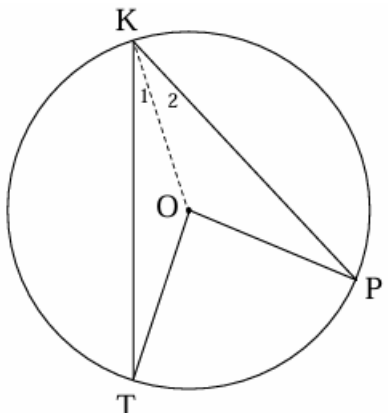
$\therefore \hat{O}_2 = 2\hat{K}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$

$= 2\left(\hat{K}_1 + \hat{K}_2\right)$

$\therefore T\hat{O}P = 2\,T\hat{K}P$

OR



9.1 Construction: Draw KO

$\hat{T} = \hat{K}_1$ [$\angle$ s opp. equal sides/*$\angle$e teenoor gelyke sye*]

$\therefore K\hat{O}T = 180° - 2\hat{K}_1$ [sum of $\angle$ s of Δ / *binne $\angle$e van* Δ]

$\hat{P} = \hat{K}_2$ [$\angle$ s opp. equal sides/*$\angle$e teenoor gelyke sye*]

$\therefore K\hat{O}P = 180° - 2\hat{K}_2$ [sum of $\angle$ s of Δ / *binne $\angle$e van* Δ]

$T\hat{O}P = 360° - \left(K\hat{O}T + K\hat{O}P\right)$ [$\angle$ s around a point/*$\angle$e om 'n punt*]

$= 360° - \left(180° - 2\hat{K}_1 + 180° - 2\hat{K}_2\right)$

$= 2\hat{K}_1 + 2\hat{K}_2$

$= 2\left(\hat{K}_1 + \hat{K}_2\right)$

$\therefore T\hat{O}P = 2\,T\hat{K}P$

9.2.1 $\hat{O}_2 = 64°$ [$\angle$ at centre = 2 $\times\angle$ at circumference/ *Midpts$\angle$ = 2 $\times$ Omtreks$\angle$*]

OR

$PLO = \hat{P} = 32°$ [$\angle$s opp equal radii/ *$\angle$e teenoor gelyke radiusse*]

$\hat{O}_2 = 64°$ [ext $\angle$ of Δ / *buite $\angle$ van* Δ]

9.2.2 $P\hat{F}M = 90°$ [tan $\perp$ diameter/*raaklyn $\perp$ middellyn*]

$\hat{M} = 26°$ [sum of $\angle$ s of Δ/*binne $\angle$e van Δ*]

10.1 $\hat{R} = 55°$ [sum of ∠s in Δ/*binne ∠e van* Δ]

$\therefore Q\hat{T}S = 55°$ [∠s in the same seg/*∠e in dieselfde segment*]

OR

$\hat{S}_2 = 35°$ [∠s in the same seg/*∠e in dieselfde segment*]

$\therefore Q\hat{T}S = 55°$ [sum of ∠s in Δ/*binne ∠e van* Δ]

10.2 $S\hat{P}Q = 125°$ [opp ∠s of cyclic quad/teenoorst. *∠e van kvh*]

$\hat{S}_1 = \hat{R} = 55°$ [given/*gegee*]

$S\hat{P}Q + \hat{S}_1 = 180°$

$\therefore PQ \parallel SR$ [co-int ∠s supp/*ko-binne ∠e suppl*]

OR

$\hat{S}_1 = \hat{R} = 55°$ [given/*gegee*]

$P\hat{Q}R = 125°$ [opp ∠s of cyclic quad/teenoorst. *∠e van kvh*]

$\therefore \hat{Q}_1 = 125° - 35° = 90°$

$\therefore \hat{Q}_1 + \hat{V}_1 = 180°$

$\therefore PQ \parallel SR$ [co-int ∠s supp/*ko-binne ∠e suppl*]

10.3 $\hat{Q}_1 = 90°$ [co-int ∠s; PQ|| SR/ *ko-binne ∠e; PQ //SR*]

$\therefore$ PT is a diameter [converse ∠ in semi-circle/
chord subtends 90°∠
omgekeerde ∠ in halwe sirkel /
koord onderspan 90° ∠]

OR

$\hat{S}_2 = 35°$ [ext ∠ of ΔSVT or sum of ∠s in Δ
buite ∠ v Δ of *binne ∠e van* Δ]

$P\hat{S}T = 90°$

$\therefore$ PT is a diameter [converse ∠ in semi-circle/
chord subtends 90°∠
omgekeerde ∠ in halwe sirkel /
koord onderspan 90° ∠]

11.1.1

$$\frac{FD}{CF} = \frac{GA}{CG}$$ [prop theorem; AD || EF/line || one side of Δ/ *eweredigheidst.; AD // EF / lyn // een sy v Δ*]

$$\frac{FD}{CF} = \frac{2}{3}$$

11.1.2

$$FD = \frac{2}{3}CF$$

$$FD = \frac{2}{3}(2x) = \frac{4}{3}x$$

$$\frac{BA}{EA} = \frac{BD}{FD}$$ [prop theorem; AD || EF/line || one side of Δ/ *eweredigheidst.; AD // EF / lyn // een sy v Δ*]

$$\frac{BA}{EA} = \frac{5x - \frac{4}{3}x}{\frac{4}{3}x}$$

$$= \frac{11}{3} \times \frac{3}{4}$$

$$= \frac{11}{4}$$

11.1.3

$$\frac{\text{Area of } \Delta GCF}{\text{Area of } GFDA} = \frac{\text{Area } \Delta GCF}{\text{Area } \Delta CDA - \text{Area } \Delta GCF}$$

$$= \frac{\frac{1}{2}GC.CF\sin\hat{C}}{\frac{1}{2}AC.CD\sin\hat{C} - \frac{1}{2}GC.CF\sin\hat{C}}$$

$$= \frac{\frac{1}{2}(3k)(3p)(\sin\hat{C})}{\frac{1}{2}(5k)(5p)(\sin\hat{C}) - \frac{1}{2}(3k)(3p)(\sin\hat{C})}$$

$$= \frac{\frac{1}{2}(9kp)(\sin\hat{C})}{\frac{1}{2}\sin\hat{C}(25kp - 9kp)}$$

$$= \frac{9}{16}$$

11.2.1	$\frac{QE}{QR}=\frac{PW}{PR}$ [prop theorem; PQ || WE/line || one side of Δ / *eweredigheidst.; PQ || WE / lyn || een sy v* Δ] $PR=\frac{PW.QR}{QE}$
11.2.2	$\frac{PQ}{RQ}=\frac{QZ}{QP}$ $[\Delta PQZ \,\|\|\| \,\Delta RQP]$ $\therefore PQ^2 = RQ.QZ$
11.2.3	In ΔQSZ and ΔQRS $\hat{Q}_2=\hat{Q}_2$ [common $\angle$ / *gemeenskaplike* $\angle$] $\hat{S}_1=\hat{R}_2$ [tan chord theorem/*raaklyn koord stelling*] $\hat{Z}_1=Q\hat{S}R$ [3rd $\angle$ of Δ] $\therefore \Delta QSZ \,\|\|\| \,\Delta QRS$ [$\angle\angle\angle$]
11.2.4	$\frac{QS}{QR}=\frac{QZ}{QS}$ $[\Delta QSZ \|\|\| \Delta QRS]$ $\therefore QS^2 = QZ.QR$ But $PQ^2 = RQ.QZ$ [proved in 11.2.2] $\therefore PQ = QS$

11.2.5

$$\frac{PQ}{RQ} = \frac{PZ}{PR} \quad [\Delta PQZ \,|||\, \Delta RQP]$$

$$PR = \frac{QR.PZ}{PQ}$$

$$PR = \frac{PW.QR}{QE} \quad \text{[proved in 11.2.1]}$$

$$\therefore \frac{PW.QR}{QE} = \frac{QR.PZ}{PQ}$$

$$PW = \frac{QE.PZ}{PQ}$$

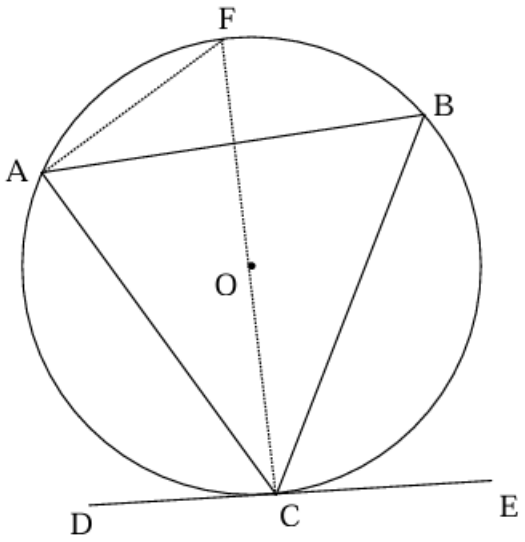
But $PQ^2 = RQ.QZ$ [proved in 11.2.2]

$$\therefore PQ = \sqrt{RQ.QZ}$$

$$\therefore PW = \frac{QE.PZ}{\sqrt{QR.QZ}}$$

Paper B

8.1



Construction: Draw diameter CF and draw AF
Konstruksie: Trek middellyn CF *en verbind* AF

$F\hat{C}E = 90°$ [tan ⊥ radius/*raaklyn ⊥ radius*]

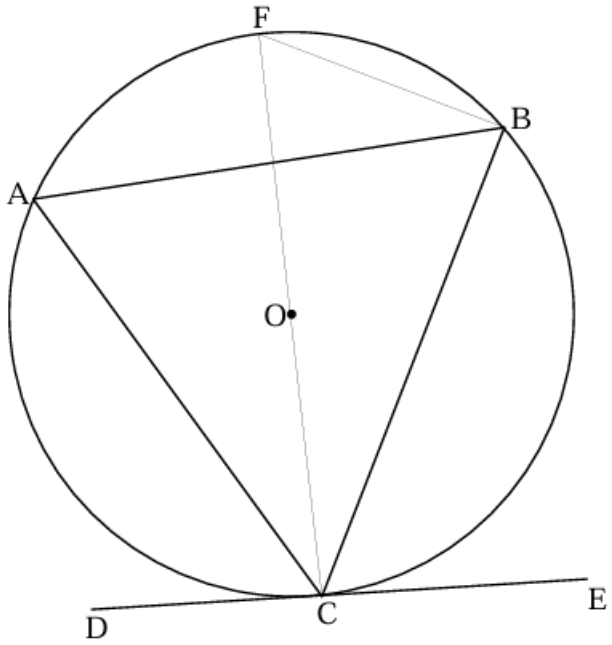
$F\hat{A}C = 90°$ [∠ in semi circle/*∠ in halwe sirkel*]

$F\hat{A}B = F\hat{C}B$ [∠s same segment/*∠e dieselfde segm*]

$\therefore B\hat{A}C = B\hat{C}E$

$\therefore B\hat{C}E = \hat{A}$

OR



Construction: Draw diameter CF and draw FB
Konstruksie: Trek middellyn CF *en verbind* FB

$F\hat{B}C = 90°$ [∠ in semi circle/*∠ in halwe sirkel*]

$B\hat{F}C + F\hat{C}B = 90°$ [sum of ∠s in Δ/*binne ∠e v Δ*]

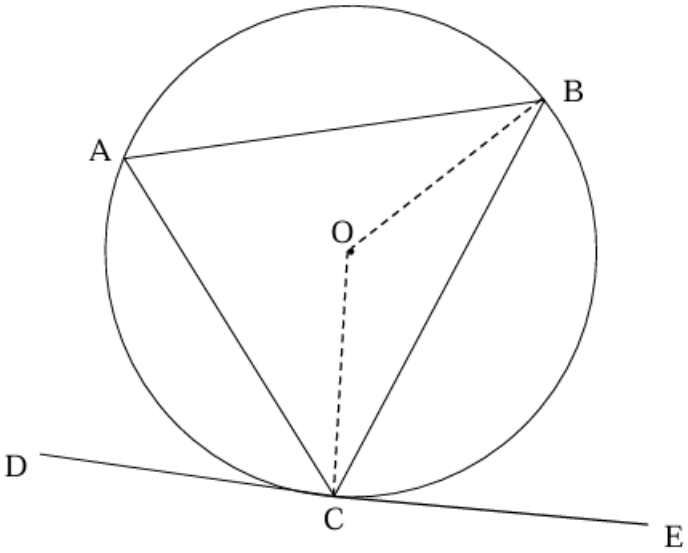
$O\hat{C}E = 90°$ [tan ⊥ radius/ *raaklyn ⊥ radius*]

$\therefore B\hat{C}E = \hat{F}$

but $\hat{A} = \hat{F}$ [∠s in same seg/*∠ in dies. segment*]

$\therefore B\hat{C}E = \hat{A}$

OR



Construction: Draw radii BO and OC
Konstruksie: Trek radiusse BO en OC

$O\hat{C}E = 90°$ or $B\hat{C}E = 90° - O\hat{C}B$ [tan $\perp$ radius / *raaklyn* $\perp$ *radius*]

$O\hat{C}B = O\hat{B}C$ [$\angle$s opp equal sides/ *$\angle$e teenoor gelyke sye*]

$\therefore C\hat{O}B = 180° - 2O\hat{C}B$ [$\angle$s of Δ/*$\angle$e van Δ*]

$C\hat{A}B = 90° - O\hat{C}B$ [$\angle$ at centre = 2×$\angle$ circumf/ *midpts$\angle$ = 2× omtreks$\angle$*]

$\therefore B\hat{C}E = C\hat{A}B$

8.2.1	$\hat{P} = \hat{R}_2 = 68°$	[ext $\angle$ of cyclic quad /*buite $\angle$ van kvh*]
8.2.2	$\hat{Q}_1 = \hat{S}_2$	[$\angle$s opp equal sides /*$\angle$e teenoor gelyke sye*]
	$\hat{Q}_1 + \hat{S}_2 = 68°$	[ext $\angle$ of Δ/ *buite $\angle$ van Δ*]
	$\therefore \hat{Q}_1 = 34°$	
8.2.3	$\hat{S}_1 = \hat{Q}_1 = 34°$	[tan-chord theorem/*$\angle$ tussen rkl en koord*]

9.1	$\hat{A}_2 = \hat{D}_1 = 37°$	[∠s in the same seg/*∠e in dies segment*]
	$\hat{A}_1 = \hat{A}_2 = 37°$	[BA bisects $C\hat{A}D$ /BA *halveer* $C\hat{A}D$]
	$\hat{D}_3 = \hat{A}_1 = 37°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]
	$\hat{C}_2 = \hat{A}_2 = 37°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]
9.2	$A\hat{D}G = 53°$	[∠ in semi circle/*∠ in halwe sirkel*]
	$\hat{A}_1 = 37°$	[proved in 9.1/*reeds bewys in 9.1*]
	$\therefore \hat{G}_1 = 90°$	[sum of ∠s in Δ/*binne ∠e van* Δ]
	$\therefore CG = DG$	[line from centre ⊥ to chord/ *lyn uit midpt.* ⊥ *op koord*]
	OR	
	$\hat{F}_2 = 2\hat{D}_1 = 74°$	[∠ at centre = 2 ×∠ at circumference/ *midpt. ∠s = 2×omtreks∠*]
	$\hat{D}_3 = 37°$	[proved in 9.1/*reeds bewys in 9.1*]
	$\therefore \hat{D}_2 = 16°$	[∠ in semi circle/*∠ in halwe sirkel*]
	$\hat{C}_1 = \hat{D}_2 = 16°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]
	$\therefore \hat{G}_2 = 90°$	[sum of ∠s in Δ/*binne ∠e van* Δ]
	$\therefore CG = DG$	[line from centre ⊥ to chord/ *lyn uit midpt.* ⊥ op *koord*]

9.3 $\hat{F}_2 = 2\hat{D}_1 = 74°$ **OR** $\hat{F}_2 = 2\hat{A}_2 = 74°$ [$\angle$ at centre = 2 $\times\angle$ at circum./ *midpt. $\angle$s = 2×omtreks$\angle$*]

$$\frac{FG}{20} = \cos 74°$$
$$FG = 5,51$$
$$\therefore BG = 14,49 \text{ units}$$

OR

$\hat{F}_2 = 2\hat{D}_1 = 74°$ [$\angle$ at centre = 2 $\times\angle$ at circumference *midpt. $\angle$ = 2×omtreks$\angle$*]

$$\frac{FG}{20} = \sin 16°$$
$$FG = 5,51$$
$$\therefore BG = 14,49 \text{ units}$$

OR

$$\frac{DG}{20} = \cos 16°$$
$$DG = 19,23$$

$$\frac{BG}{19,23} = \tan 37°$$
$$BG = 14,49 \text{ units}$$

OR

$$\frac{DG}{20} = \cos 16°$$
$$DG = 19,23$$

$FG^2 = FD^2 - DG^2$ [Pythagoras]
$FG^2 = 20^2 - (19,23)^2$
$FG = 5,51$

$BG = 20 - 5,51$
$= 14,49$ units

10.1	$\hat{O}_1 = 90°$	[given/*gegee*]
	$\hat{F}_2 + \hat{F}_3 = 90°$	[$\angle$ in semi circle/$\angle$ *in halwe sirkel*]
	$\hat{O}_1 = \hat{F}_2 + \hat{F}_3 = 90°$	
	$\therefore$ TODF is a cyclic quad	[ext $\angle$ = int opp $\angle$/ *buite* $\angle$ *= teenoorst. binne* $\angle$] **OR** [converse ext $\angle$ of cyclic quad/ *omgekeerde buite* $\angle$ *v kvh*]
10.2	$\hat{T}_1 = \hat{T}_3$	[vert opp $\angle$s =/ *regoorstaande* $\angle$*e*]
	But $\hat{D}_3 = \hat{T}_3$	[ext $\angle$ of cyclic quad/ *buite* $\angle$ *v kvh*]
	$\therefore \hat{T}_1 = \hat{D}_3$	
10.3	In ΔDFE and ΔTFO	
	1) $\hat{D}_3 = \hat{T}_3$	[ext $\angle$ of cyclic quad/ *buite* $\angle$ *v kvh*]
	2) $\hat{F}_4 = \hat{C}_2$	[tan-chord theorem/ $\angle$ *tussen rkl en koord*]
	but $\hat{C}_2 = \hat{F}_2$	[$\angle$s opp equal sides/ $\angle$*e teenoor gelyke sye*]
	$\therefore \hat{F}_4 = \hat{F}_2$	
	3) $\hat{E} = \hat{O}_2$	[3rd $\angle$ of Δ/$\angle$*e van* Δ]
	ΔTFO ||| ΔDFE	[$\angle\angle\angle$]

	OR	
	In ΔDFE and ΔTFO	
	1) $\hat{D}_3 = \hat{T}_3$	[ext ∠ of cyclic quad/*buite ∠ van Δ*]
	2) $\hat{F}_4 = \hat{C}_2$	[tan-chord theorem/*∠ tussen rkl en koord*]
	$\hat{F}_2 + \hat{F}_3 = 90°$	[∠ in semi circle/*∠ in halwe sirkel*]
	$\hat{D}_1 + \hat{D}_2 = 90° - \hat{C}_2$	[sum of ∠s in Δ/ *binne ∠e van Δ*]
	$\hat{E} = 90° - 2\hat{F}_4$	[ext ∠ of Δ/ *buite ∠ van Δ*]
	$\hat{O}_3 = 2\hat{C}_2$	[∠ at centre = 2 ×∠ at circumference/ *midpt. ∠s = 2×omtreks∠*]
	$\hat{O}_2 = 90° - 2\hat{F}_4$	[∠s on a str line/*∠e op 'n reguitlyn*]
	$\hat{O}_2 = \hat{E}$	
	3) $\therefore \hat{F}_4 = \hat{F}_2$	[3rd ∠ of Δ/*∠e van Δ*]
	ΔTFO ||| ΔDFE	[∠∠∠]
10.4	$\hat{B}_2 = \hat{D}_1$	[∠s opp equal sides/*∠e teenoor gelyke sye*]
	$\hat{B}_2 = \hat{E}$	[given/*gegee*]
	$\therefore \hat{D}_1 = \hat{E}$	
	$\therefore$ DB ‖ EA	[corresp ∠s =/*ooreenkomstige ∠e gelyk*]
10.5	In ΔOEA	
	DB ‖ EA	[proven/*reeds bewys*]
	$\frac{OD}{DE} = \frac{OB}{BA}$	[line ‖ one side of Δ/*lyn // een sy van Δ*]
		OR
		[prop theorem; DB ‖ EA/ *eweredigheid stelling; DB // EA*]
	$\therefore DE = \frac{DO.AB}{OB}$	
	$\frac{FO}{FE} = \frac{TO}{DE}$	[ΔTFO ||| ΔDFE]
	$DE = \frac{TO.FE}{FO}$	
	$\therefore \frac{DO.AB}{OB} = \frac{TO.FE}{FO}$	
	$\therefore \frac{DO.AB}{DO} = \frac{TO.FE}{DO}$	[DO = OB = FO]
	$\therefore DO = \frac{TO.FE}{AB}$	

Paper C

8.1.1(a)	$\hat{T}_2 = 54°$	[tan ⊥ rad]
8.1.1(b)	$\hat{L}=36°$	[tan - chord theorem]
8.1.1(c)	$K\hat{O}T=72°$	[∠ at centre = 2×∠ at circumference]
	OR/*OF*	
	$O\hat{K}T=\hat{T}_2=54°$	[∠s opposite = radii]
	$K\hat{O}T=180°-(54°+54°)$	[sum of int ∠'s of Δ]
	$=72°$	
8.1.2	$K\hat{M}O=180°-(18°+72°)$	
	$=90°$	[sum of int ∠'s of Δ]
	∴ KM = ML	[line from centre ⊥ to chord]
	OR/*OF*	
	$O\hat{K}T=54°$	[∠s opposite = radii]
	$\hat{K}_1=54°-18°=36°$	
	$T\hat{M}K=90°$	[sum of int ∠'s of Δ]
	∴ KM = ML	[line from centre ⊥ to chord]

8.2.1

$$\frac{DC}{CS}=\frac{20}{12}=\frac{5}{3}$$

$$\therefore \frac{DC}{CS}=\frac{RB}{BS}$$

$\therefore$ BC $\|$ DR [converse line $\|$ one side of Δ OR sides in the

$\therefore$ BC $\|$ AD same proportion]

8.2.2

$$\frac{AR}{AD}=\frac{RB}{BS}$$ [line $\|$ one side of Δ] **OR**[Prop Theorem AB $\|$ DS]

$$\frac{AR}{AD}=\frac{5}{3}$$

$$\frac{48-AD}{AD}=\frac{5}{3}$$

$\therefore 5AD = 144-3AD$

$AD = 18$

AB = 20 [opp sides of parm]

$\therefore$ AD : AB = 18 : 20 = 9 : 10

OR/OF

$$\frac{AR}{RD}=\frac{5}{8}\text{.....prop thm AB}\|\text{DS}$$

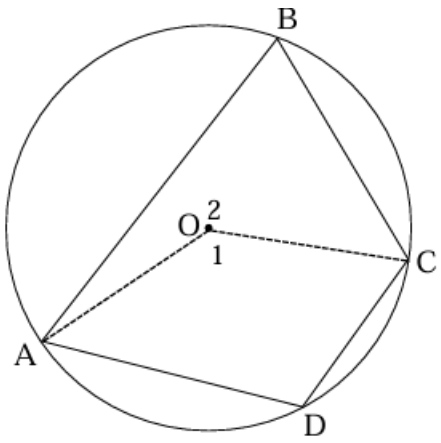
$$\frac{AR}{48}=\frac{5}{8}$$

$$\therefore AR=30 \text{ and } AD=18$$

$$\therefore \frac{AR}{RD}=\frac{AB}{DS}\text{.....}|||\Delta's$$

$$\therefore AB=20$$

$$\therefore AB:AD=18:20=9:10$$



9.1 Constr: Draw radii OA and OC.

Proof:

$\hat{O}_1 = 2\hat{B}$ [$\angle$ at centre $= 2\times\angle$ at circumference]

$\hat{O}_2 = 2\hat{D}$ [$\angle$ at centre $= 2\times\angle$ at circumference]

$\hat{O}_1 + \hat{O}_2 = 360°$ [revolution]

$2\hat{B} + 2\hat{D} = 360°$ [revolution]

$\therefore \hat{B} + \hat{D} = 180°$

9.2

$E\hat{F}G = 180° - \hat{D}_1$ [opp $\angle$'s of cyclic quad]

$\therefore E\hat{F}G = 180° - x$

$E\hat{F}G = 180° - \hat{G}$ [co-int $\angle$'s; EF || DG]

$\hat{G} = x$

But $\hat{G} = \hat{D}_2$ [alt $\angle$'s; DH || FG]

$\therefore \hat{D}_1 = \hat{D}_2 = x$

10.1.1

$T\hat{P}R = 90°$	[$\angle$ in semi-circle]
$S\hat{P}R = 90°$	[$\angle$'s on a straight line]
$\therefore$ SR is a diameter	[converse $\angle$ in semi-circle]

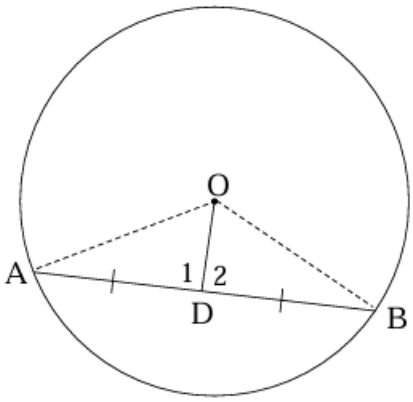
OR

$T\hat{K}R = 90°$	[$\angle$ in semi-circle]
$S\hat{P}R = 90°$	[ext $\angle$ of cyclic quad]
$\therefore$ SR is a diameter	[converse $\angle$ in semi-circle]
	OR
	[chord subtends a right angle]

10.1.2	$\hat{R}_1 = P\hat{T}K$ [ext $\angle$ of cyclic quad] $\hat{P}_1 = P\hat{T}K = \hat{R}_1$ [$\angle$s opp equal sides] $\hat{S} + \hat{R}_1 = \hat{P}_1 + P_2$ [ext $\angle$ of Δ] $\therefore \hat{S} = \hat{P}_2$ [$\hat{R}_1 = \hat{P}_1$]
10.1.3	In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $\Delta SPK \,\|\|\| \,\Delta PRK$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $S\hat{P}K = P\hat{R}K$ [sum of $\angle$s in Δ] $\Delta SPK \,\|\|\| \,\Delta PRK$
10.2	$\frac{PK}{RK} = \frac{SK}{PK}$ [$\Delta SPK \,\|\|\| \,\Delta PRK$] $PK^2 = SK.RK$ $ST^2 = SK^2 + TK^2$ [Pythagoras] $TK = PK$ [Given] $ST^2 = SK^2 + PK^2$ $ST^2 = SK^2 + SK.RK$ $ST^2 = (2RK)^2 + 2RK.RK$ $ST^2 = 6RK^2$ $ST = \sqrt{6}RK$

Paper D

8.1.1	$\hat{P} = 116°$	[opp ∠s of cyclic quad/*teenoorst. ∠e van kvh*]
8.1.2	$\hat{M}_1 + 64° = 90°$ $\hat{M}_1 = 26°$	[∠ in semi-circle/∠ *in halwe sirkel*]
8.1.3	$\hat{O}_1 = 52°$	[∠ at centre = 2 x ∠ at circumference/*midpts. ∠ = 2 x omtreks. ∠*]
8.2.1	Midpt theorem/*Midpt. Stelling* **OR/*OF*** Converse prop intercept theorem	
8.2.2	BG = 2DE or $6x - 2$ $BG = 6x - 2$ $\frac{GH}{BG} = \frac{FC}{BF}$ $\frac{x+1}{6x-2} = \frac{1}{4}$ $4x + 4 = 6x - 2$ $2x = 6$ $x = 3$	[Midpt theorem/*Midpt. stelling*] [line ‖ one side of Δ **OR** prop theorem; FG ‖ CH / *lyn ‖ een sy v. Δ*]
	OR	
	$\frac{BF}{FC} = \frac{BG}{GH}$ $\frac{AE}{AG} = \frac{DE}{BG}$ $BG = 4x + 4$ $\frac{1}{2} = \frac{3x-1}{4x+4}$ $\therefore 4x + 4 = 6x - 2$ $\therefore x = 3$	[line ‖ one side of Δ **OR** prop theorem; FG ‖ CH / *lyn ‖ een sy v. Δ*] [ΔADE ||| ΔABG]



9.1.1

Construction:
Draw OA and OB
In ΔADO and ΔBDO

OA = OB	[radii/*radiusse*]
OD = OD	[common side/*gemeenskaplike sy*]
AD = DB	[given/*gegee*]
$\therefore \Delta ADO \equiv \Delta BDO$	[S;S;S]

ADB is a straight line

$\therefore \hat{D}_1 = \hat{D}_2$	$\Delta ADO \equiv \Delta BDO$
$\therefore OD \perp AB$	[∠s on a str line/*∠e op 'n reguitlyn*]

OR/*OF*

Construction:
Draw OA and OB
In ΔADO and ΔBDO

AD = DB	[given/*gegee*]
$\hat{A} = \hat{B}$	[∠s opp; ∠s sides /*∠e teenoor gelyke sye*]
OA = OB	[radii/*radiusse*]
$\therefore \Delta ADO \equiv \Delta BDO$	[S;∠;S]

ADB is a straight line

$\therefore \hat{D}_1 = \hat{D}_2$	$\Delta ADO \equiv \Delta BDO$
$\therefore OD \perp AB$	[∠s on a str line/*∠e op 'n reguitlyn*]

9.2.1	$O\hat{T}G = 90°$	[line from centre to midpt of chord/ *midpt. sirkel; midpt. koord*]
	$O\hat{B}G = 90°$	[tan ⊥ radius/*raaklyn ⊥ radius*]
	$\therefore O\hat{T}G = O\hat{B}G = 90°$	
	$\therefore$ OTBG is a cyclic quadrilateral	[line subtends equal ∠s **OR** converse ∠s in the same segment/ *lyn onderspan gelyke ∠e*]
9.2.2	$\hat{S} = B\hat{T}G$	[corresp ∠s; GF ‖ PS / *ooreenk. ∠s; GF ‖ PS*]
	But $B\hat{T}G = G\hat{O}B$	[∠s in the same segment/*∠e in dies. sirkelsegment*]
	$G\hat{O}B = \hat{S}$	
10.1	$\hat{P}_1 = \hat{Q}_1$	[tan-chord theorem/*∠ tussen raaklyn en koord*]
	$\hat{S}_1 = \hat{Q}_1 + \hat{Q}_2$	[ext ∠ of cyclic quad/*buite ∠ v. kvh*]
	$\therefore \hat{S}_1 = \hat{P}_1 + \hat{Q}_2$	
	$\hat{T}_2 = \hat{R}_2 + \hat{Q}_2$	[ext ∠ of Δ/*buite ∠ v. Δ*]
	but $\hat{P}_1 = \hat{R}_2$	[given/*gegee*]
	$\hat{T}_2 = \hat{P}_1 + \hat{Q}_2$	
	$\therefore \hat{S}_1 = \hat{T}_2 = \hat{P}_1 + \hat{Q}_2$	
10.2	In Δ ASD and ΔACR	
	$\hat{A} = \hat{A}$	[common ∠/*gemeenskaplike ∠*]
	$\hat{S}_1 = \hat{T}_2$	[proven/*reeds bewys*]
	$\hat{T}_2 = \hat{C}_2$	[alt ∠s; QS ‖ CA/verw. *∠e*; QS ‖ CA]
	$\therefore \hat{S}_1 = \hat{C}_2$	
	$\hat{D}_1 = \hat{R}_1$	[sum of ∠s in Δ/*∠e v. Δ*]
	ΔASD ||| ΔACR	
	$\therefore \frac{AD}{AR} = \frac{AS}{AC}$	[corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]

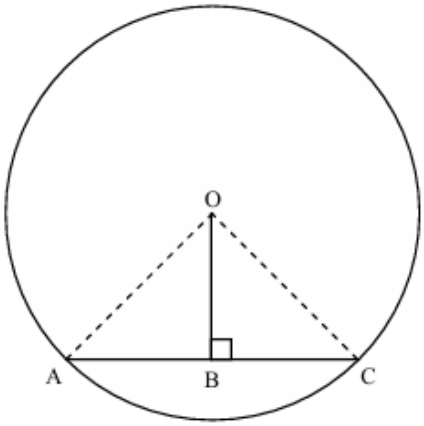
OR

	In Δ ASD and ΔACR	
	$\hat{A} = \hat{A}$	[common $\angle$/*gemeenskaplike* $\angle$]
	$\hat{S}_1 = \hat{T}_2$	[proven/*gegee*]
	$\hat{T}_2 = \hat{C}_2$	[alt $\angle$s; QS $\parallel$ CA/verw. $\angle e$; QS $\parallel$ CA]
	$\therefore \hat{S}_1 = \hat{C}_2$	
	$\Delta ASD \,\|\|\| \,\Delta ACR$	[$\angle;\angle;\angle$]
	$\therefore \frac{AD}{AR} = \frac{AS}{AC}$	[corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]
10.3	$\frac{AS}{AC} = \frac{SD}{CR}$	[$\Delta ASD \,\|\|\| \,\Delta ACR$]
	$\therefore AS = \frac{AC \times SD}{CR}$	
	$\frac{AS}{AR} = \frac{CT}{CR}$	[line $\parallel$ one side of Δ OR prop theorem; TS $\parallel$ CA/*lyn* $\parallel$ *een sy v.* Δ]
	$\therefore AS = \frac{AR \times CT}{CR}$	
	$\therefore \frac{AC \times SD}{CR} = \frac{AR \times CT}{CR}$	
	$\therefore AC \times SD = AR \times CT$	

Paper E

9.1	AF : FE = 2 : 1	(Prop Th; FB || EC) (*Eweredigheid St;* FB || EC)
9.2	$\frac{AF}{FE} = \frac{2}{1}$ $FE = \frac{AF}{2} = \frac{8}{2} = 4\text{ cm}$ AE = 12 cm $\frac{ED}{AE} = \frac{1}{2}$ $\frac{ED}{12} = \frac{1}{2}$ ED = 6 cm	(BE || DC; Prop Th) / *(BE || DC; Eweredigheid St)*

10.1	$\hat{E}_2 = 90° - 56°$ (∠s in a semi circle) $\hat{E}_2 = 34°$ *(∠e in 'n halwe sirkel)*
10.2	$C\hat{E}H = \hat{E}_1 + \hat{E}_2 = 66°$ $\therefore E\hat{B}C = \hat{B}_2 + \hat{B}_3 = 66°$ (tan ch th) *(raaklyn koord)* **OR** $\hat{B}_2 = 32°$ (tan ch th) *(raaklyn koord)* $\hat{B}_3 = 34°$ (∠s in the same segment) *(∠e in selfde seg)* $E\hat{B}C = 66°$
10.3	$\hat{E}_1 + \hat{E}_2 + \hat{E}_3 = 122°$ $\hat{F} = 122°$ (tan ch th) / *(raaklyn koord)* **OR** $\hat{C} = 58°$ (sum of int ∠s of Δ) / *(som van binne∠e Δ)* $\hat{F} = 122°$ (opp ∠s cyclic quad) / *(oorst ∠ koordevierhoek)* **OR** $\hat{D} = 58°$ (sum of int ∠s of Δ) / *(som van binne∠e Δ)* $\hat{F} = 122°$ (opp ∠s cyclic quad) / *(oorst ∠ koordevierhoek)*



Construct radii OA and OC.
In ΔOAB and ΔOCB

i. OB is common
ii. OA = OC (radii)
iii. $O\hat{B}A = O\hat{B}C = 90°$ (given)

ΔOAB ≡ ΔOCB (90°HS)
AB = BC (≡ Δs)

OR

Construct radii OA and OC.
In ΔOAB and ΔOCB

i. OB is common **OR** OA = OC (radii)
ii. $\hat{A} = \hat{C}$ (∠s opp = radii)
iii. $O\hat{B}A = O\hat{B}C = 90°$ (given)

ΔOAB ≡ ΔOCB (SAA)
AB = BC (≡ Δs)

OR

Construct radii OA and OC.

$O\hat{B}A = O\hat{B}C = 90°$ (given)
$OA^2 = OB^2 + AB^2$ (Pythagoras)
$OC^2 = OB^2 + BC^2$ (Pythagoras)
OC = OA (radii)
OB is common
$AB^2 = BC^2$
AB = BC

12.1	$\hat{K}_3 = x$	(tan ch th) *(raaklyn koord)*
	$\hat{Y}_2 = x$	(∠s in same seg) *(∠e in selfde seg)*
	$\hat{K}_2 = x$	(∠s opp = radii) / (= chs subt = ∠s) *(∠e oork = radiusse) / (= koorde = ∠e)*
	$\hat{W}_2 = x$	(∠s in same seg) / (= chs subt = ∠s) *(∠e in selfde seg) / (= koorde = ∠e)*
12.2	$\hat{O}_1 + \hat{O}_4 = 180° - 2x$	(sum of int ∠'s of Δ) / (opp∠ cyclic quad) *(som van binne∠e Δ) / (oorst ∠ koordevierhoek)*
	$\hat{T} = 90° - x$	(∠ at circ cent = 2∠ at circumference)

12.3

$\hat{E}_2 = 90°$ (sum of int ∠'s of Δ) / *(som van binne∠e Δ)*

KE = ET (⊥ from centre to chord bisects chord)
(⊥ van middelpunt tot koord halveer koord)

OR

$\hat{K}_1 + \hat{K}_2 + \hat{K}_3 = 90° - x$ (sum of int ∠'s of Δ)

In ΔKWE and ΔTEW

1. $\hat{K}_1 + \hat{K}_2 + \hat{K}_3 = \hat{T} = 90° - x$ (proven above)
2. $\hat{W}_1 = \hat{W}_2 = x$ (Proven in 12.1)
3. WE is common

$\therefore$ ΔKWE ≡ ΔTEW (∠∠S)

KE = ET

12.4

In ΔKOE and ΔWTE

i. $\hat{K}_3 = \hat{W}_2 = x$ (proven)

ii. $\hat{E}_2 = \hat{E}_1 = 90°$ (∠s on str line / sum of int ∠'s of Δ)
(∠e op reguit lyn / som van binne∠e Δ)

iii. $\hat{O}_2 = \hat{T} = 90° - x$ (3^{rd} ∠ of Δ)

ΔKOE ||| ΔWTE (∠∠∠)

$\frac{KE}{WE} = \frac{OE}{TE}$ (||| Δs)

KE = TE (proven)

KE.TE = OE.WE

$KE^2 = OE.WE$

OR

In ΔKOE and ΔKWE

i. $\hat{K}_3 = \hat{W}_1 = x$ (proven)

ii. $\hat{E}_1$ is common

iii. $\hat{O}_2 = \hat{K}_1 + \hat{K}_2 + \hat{K}_3$ (3^{rd} ∠ of Δ)

ΔKOE ||| ΔWKE (∠∠∠)

$\frac{KE}{WE} = \frac{OE}{KE}$ (||| Δs)

$KE^2 = OE.WE$

Paper F

8.1.1(a)	$\hat{O}_2 = 64°$ [∠ at centre = 2 ×∠ at circumference/ *Middelpts ∠ = 2 ×∠ omtreks∠*]
8.1.1(b)	$\hat{M}_2 = 90°$ [Line from centre to midpt of chord/*lyn v midpt na midpt v koord*] $K\hat{O}N = 90° + 26° = 116°$ [ext ∠ of Δ/*buite ∠ van* Δ] $\hat{O}_1 = 116° - 64° = 52°$ **OR** $\hat{M}_2 = 90°$ [Line from centre to midpt of chord/*lyn v midpt na midpt v koord*] $\hat{O}_3 = 64°$ [sum of ∠s in Δ] $\hat{O}_1 = 52°$ [∠s on straight line/*op 'n reguitlyn*]
8.1.2	$P\hat{K}O + \hat{P} = 128°$ [sum of ∠s in Δ/*som ∠e van* Δ] $P\hat{K}O = \hat{P}$ [∠s opp = sides/*∠e teenoor = sye*] $= 64°$ $\therefore \hat{K}_2 = 32°$ *or* $\hat{K}_2 = \hat{K}_1$ $\therefore$ KN bisects/*halveer* $O\hat{K}P$ **OR** $\hat{K}_2 = K\hat{N}O$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{K}_2 + K\hat{N}O = 64°$ [sum of ∠s in Δ/*som ∠e van* Δ] $\therefore \hat{K}_2 = 32°$ *or* $\hat{K}_2 = \hat{K}_1$ $\therefore$ KN bisects/*halveer* $O\hat{K}P$

8.2.1

$\hat{F}_1 = \hat{D}_1$ [tan chord theorem/*raaklyn koordst*]

$\hat{D}_1 = \hat{B}$ [Given/*Gegee*]

$\therefore \hat{F}_1 = \hat{B}$

$\therefore FG \parallel BC$ [corresp $\angle$s =/*Ooreenkomstige* $\angle e$ =]

8.2.2

$$\frac{GC}{AC} = \frac{FB}{AB}$$ [line || one side of Δ/*lyn // een sy v* Δ]

$$\frac{x+9}{2x-6} = \frac{5}{7}$$

$$7x + 63 = 10x - 30$$

$$3x = 93$$

$$x = 31$$

OR

$$AG = 2x - 6 - (x+9) = x - 15$$

$$\frac{AG}{GC} = \frac{AF}{FB}$$ [line || one side of Δ/*lyn // een sy v* Δ]

$$\frac{x-15}{x+9} = \frac{2}{5}$$

$$5x - 75 = 2x + 18$$

$$3x = 93$$

$$x = 31$$

OR

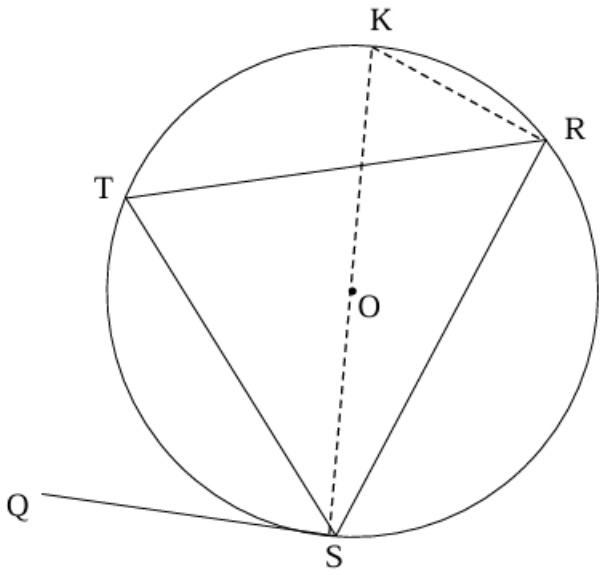
$$\frac{AF}{AB} = \frac{AG}{AC}$$ [line || one side of Δ /lyn || een sy vΔ]

$$\frac{2}{7} = \frac{x-15}{2x-6}$$

$$7x - 105 = 4x - 12$$

$$3x = 93$$

$$x = 31$$



9.1 Construction: Draw diameter KS and draw KR
Konstruksie: Trek middellyn KS en verbind KR

$\hat{QST} = 90° - \hat{TSK}$ [radius $\perp$ tangent/*raaklyn*]

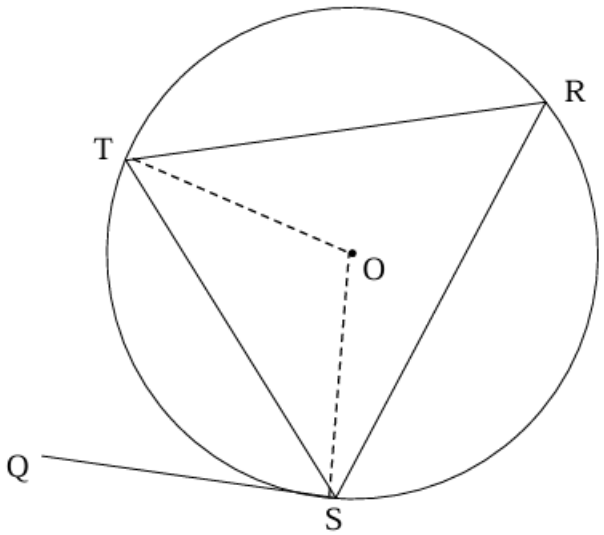
$\hat{SRK} = 90°$ [$\angle$ in semi circle/*halfsirkel*]

$\therefore \hat{SRT} = 90° - \hat{KRT}$

$\hat{TSK} = \hat{TRK}$ [$\angle$s same segment/$\angle$*e dieselfde segment*]

$\therefore \hat{QST} = \hat{R}$

OR



9.1 Construction: Draw radii OS and OT
Konstruksie: Trek radii OS en OT

$\hat{QST} = 90° - \hat{OST}$ [radius ⊥ tangent/*raaklyn*]

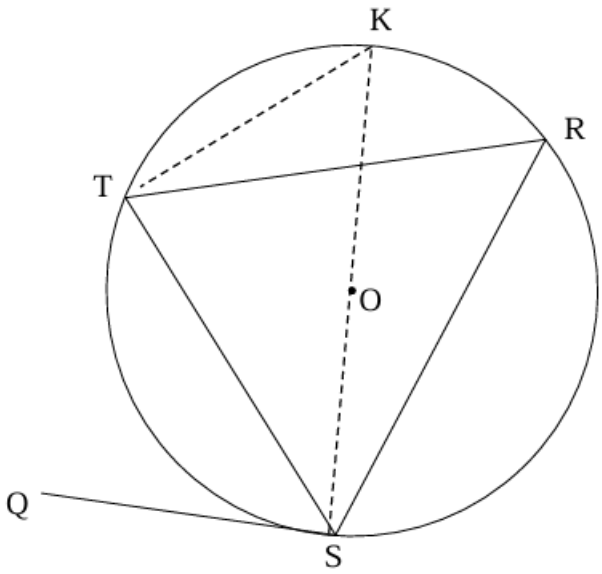
$\hat{OST} = \hat{STO}$ [∠s opp = sides/*∠e teenoor = sye*]

$\therefore \hat{SOT} = 180° - 2\hat{OST}$ [∠s of Δ/*∠e van Δ*]

$\hat{R} = 90° - \hat{OST}$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*]

$\therefore \hat{QST} = \hat{R}$

OR



9.1 Construction: Draw diameter KS and join K to T.
Konstruksie: Trek middellyn KS en verbind K tot T

$\hat{QST} = 90° - \hat{TSK}$ [radius ⊥ tangent/*raaklyn*]

$\hat{STK} = 90°$ [∠ in semi circle/*halfsirkel*]

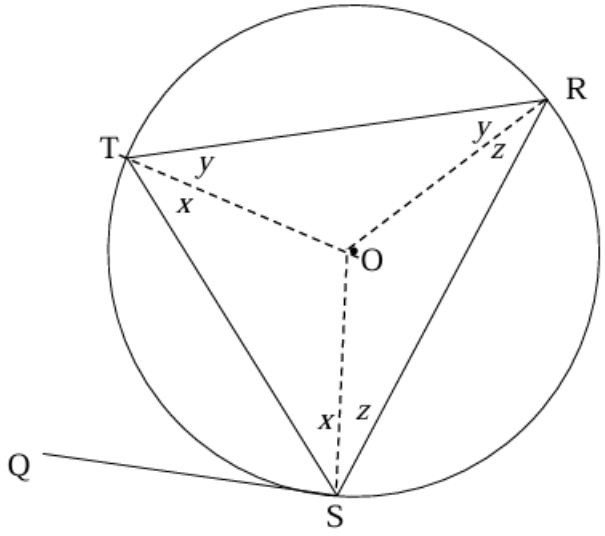
$\therefore \hat{K} = 90° - \hat{TSK}$

$\therefore \hat{QST} = \hat{K}$

but $\hat{R} = \hat{K}$ [∠s same segment/*∠e dieselfde segment*]

$\therefore \hat{QST} = \hat{R}$

OR



9.1 Construction: Draw radii OT, OR and OS
Konstruksie: Trek radiuse OT, OR en OS

$O\hat{S}T = O\hat{T}S$ [∠s opp = radii/*∠e teenoor = radiuse*]

Also: $O\hat{T}R = O\hat{R}T$ and $O\hat{R}S = O\hat{S}R$

$2x + 2y + 2z = 180°$ [∠s of Δ]

$x + y + z = 90°$

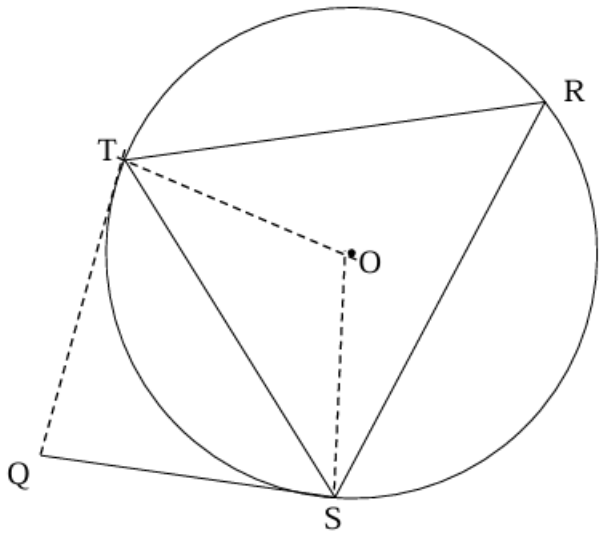
$y + z = 90° - x$

$O\hat{S}Q = 90°$ [radius ⊥ tangent/*raaklyn*]

$\therefore\ T\hat{S}Q = 90° - x$

$\therefore\ T\hat{S}Q = y + z = \hat{R}$

OR



9.1	Construction: Draw radii OT and OS, tangent QT *Konstruksie: Trek radiuse OT en OS, raaklyn QT* $O\hat{S}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore\ T\hat{S}Q = 90° - T\hat{S}O$ $\therefore\ T\hat{S}O = S\hat{T}O$ [∠s opp = radii/*∠e teenoor = radiuse*] $T\hat{O}S = 180° - 2T\hat{S}O$ [∠s of Δ] $\hat{R} = 90° - T\hat{S}O$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore\ T\hat{S}Q = \hat{R}$
9.2.1(a)	$\hat{N}_2 = x$ [alt ∠s; PR ∥ NQ/*verw. ∠e;* PR ∥ NQ]
9.2.1(b)	$\hat{Q}_2 = x$ [tan chord theorem/*raaklyn koordstelling*] OR $M_2 = x$ [tan chord theorem/*raaklyn koordstelling*] $\hat{Q}_2 = x$ [∠s in same segment/*∠e in dieselfde segm*]
9.2.2	$\frac{MN}{NR} = \frac{MS}{SP}$ [QN ∥ PR; Prop Th] $\hat{N}_1 = \hat{N}_2 = x$ [given] $\hat{P}_3 = x$ [∠s in same segment/*∠e in dieselfde segm*] $\hat{P}_3 = \hat{Q}_2$ [= *x*] $SQ = PS$ [sides opp = ∠/*sye teenoor = ∠e*] $\frac{MN}{NR} = \frac{MS}{SQ}$
10.1.1	$D\hat{B}E = 90°$ [∠ in semi-circle/*∠ in halfsirkel*] $\therefore D\hat{M}A = 90°$ [AM ⊥ DE] ∴ FBDM is a cyclic quadrilateral/*koordevh* [converse opp ∠s cyclic quad/*omgek teenoorst ∠e kvh*] **OR** $D\hat{B}E = 90°$ [∠ in semi-circle/*∠ in halfsirkel*] $\hat{M}_2 = D\hat{B}E = 90°$ ∴ FBDM is a cyclic quadrilateral/*koordevh* [converse ext∠ of cyclic quad/*omgek buite∠ van kvh*]

10.1.2	$\hat{B}_3 = \hat{D}_2$ [tangent chord th/*raaklyn koordst*] $\hat{F}_1 = \hat{D}_2$ [ext ∠ cyc quad/*buite ∠ koordevh*] $\therefore \hat{B}_3 = \hat{F}_1$ **OR** $\hat{B}_1 = \hat{E} = x$ [tangent chord th/*raaklyn koordst*] $\hat{F}_1 = 90° - x$ [∠ sum in Δ/∠ *van* Δ] $\hat{D}_2 = 90° - x$ [∠ sum in Δ/∠ *van* Δ] $\therefore \hat{F}_1 = D_2$ $\hat{B}_3 = \hat{D}_2$ [tangent chord th/*raaklyn koordst*] $\therefore \hat{B}_3 = \hat{F}_1$ **OR** $\hat{B}_1 = \hat{E} = x$ [tangent chord th/*raaklyn koordst*] $\hat{B}_3 = 90° - x$ [straight line/*reguitlyn*] $\hat{F}_1 = 90° - x$ [sum of ∠s Δ/ *som van ∠e van* Δ] $\therefore \hat{B}_3 = \hat{F}_1$
10.1.3	In ΔCDB and ΔCBE $\hat{C} = \hat{C}$ [common ∠/*gemeenskaplike ∠*] $C\hat{B}D = C\hat{E}B$ [tangent chord th/*raaklyn koordst*] $C\hat{D}B = C\hat{B}E$ [∠ sum in Δ/∠ *van* Δ] ΔCDB ||| ΔCBE **OR** In ΔCDB and ΔCBE $C\hat{B}D = C\hat{E}B$ [tangent chord th/*raaklyn koordst*] $\hat{C} = \hat{C}$ [common ∠/*gemeenskaplike ∠*] ΔCDB ||| ΔCBE [∠, ∠, ∠]
10.2.1	$\frac{BC}{EC} = \frac{DC}{BC}$ [||| Δs] $BC^2 = EC \times DC$ $= 8 \times 2$ $= 16$ $BC = 4$

10.2.2

$$\frac{BC}{EC} = \frac{DB}{BE} \quad [\|\| \ \Delta s]$$

$$\frac{DB}{BE} = \frac{4}{8} = \frac{1}{2}$$

$$BE = 2DB$$

$$DB^2 + BE^2 = DE^2 \quad [\text{Pyth theorem}]$$

$$DB^2 + (2DB)^2 = 36$$

$$5DB^2 = 36$$

$$DB^2 = \frac{36}{5}$$

$$DB = \frac{6}{\sqrt{5}} = 2{,}68 \text{ units}$$

Paper G

8.1.1 $\hat{R} = 80°$ [co-int ∠s/*ko-binne ∠e*; QW || RK]

8.1.2 $\hat{P} = 100°$ [opp ∠s of cyclic quad/*teenoorst ∠e v koordevh*]

8.1.3 $P\hat{Q}R = 136°$ [ext ∠ of cyclic quad/*buite ∠ v koordevh*]

$\hat{Q}_2 = 36°$

OR

$P\hat{U}W = \hat{S}_2 = 136°$ [corresp ∠s/*ooreenkomstige ∠e*; QW || RK]

$P\hat{Q}W + \hat{P} = P\hat{U}W$ [ext ∠s of/*buite ∠ van* Δ QPU]

$P\hat{Q}W + 100° = 136°$

$P\hat{Q}W = 36°$

OR

$\hat{U}_3 = 180° - 136° = 44°$ [co-int ∠s/*ko-binne ∠e*; QW || RK]

$\hat{U}_1 = \hat{U}_3 = 44°$ [vert opp ∠s/*regoorstaande ∠e*]

$P\hat{Q}W = 180° - (100 + 44°)$ [sum of ∠s in Δ/*som ∠e van* Δ]

$P\hat{Q}W = 36°$

8.1.4

$\hat{U}_2 = \hat{S}_2 = 136°$ [alt ∠s/*verwiss ∠e* ; QW || RK]

OR

$\hat{U}_2 = 100° + 36°$ [ext ∠s of/*buite ∠ van* ΔQPU]
$= 136°$

OR

$\hat{U}_2 = P\hat{U}W = 136°$ [vert opp ∠s/*regoorstaande ∠e*]

OR

$\hat{U}_2 = 180° - \hat{U}_3$ [∠s on a str line/*∠e op reguitlyn*]
$= 180° - 44°$
$= 136°$

8.2.1

In ΔEFT and ΔDCT:

$\frac{EF}{CD} = \frac{9}{18} = \frac{1}{2}$

$\frac{FT}{TC} = \frac{5}{10} = \frac{1}{2}$

$\frac{ET}{TD} = \frac{7}{14} = \frac{1}{2}$

$\therefore$ ΔEFT ||| ΔDCT [Sides of Δ in prop/ *sye van Δ in dieselfde verh*]

$\therefore E\hat{F}D = E\hat{C}D$

OR

In ΔFET:

$49 = 25 + 81 - 2(5)(9)\cos\hat{F}$

$\cos\hat{F} = \frac{19}{30}$

$\hat{F} = 50,7°$

In ΔTDC:

$196 = 100 + 256 - 2(10)(18)\cos\hat{C}$

$\cos\hat{C} = \frac{19}{30}$

$\hat{C} = 50,7°$

8.2.2

$E\hat{F}D = E\hat{C}D$ [proved in 8.2.1]

E, F, C and D are concyclic

EFCD is a cyclic quad [converse ∠s in the same segment/ *omgekeerde ∠e in dies segment*]

$\therefore D\hat{F}C = D\hat{E}C$ [∠s in the same segment/*∠e in dies segment*]

$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om ´n punt*]

$\therefore \hat{M} = 180° - \frac{1}{2}x$ [∠ at centre = 2 ×∠ at circumference/
middelpunts∠ = 2 × omtreks∠]

$\therefore \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ [sum of ∠s in Δ/*som ∠e van Δ*]

$\therefore \hat{T}_2 = \hat{P}_1 = \frac{1}{4}x$ [∠s opp equal sides/*∠e teenoor gelyke sye*]

$\therefore S\hat{T}M = \hat{P}_1 = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]

OR/*OF*

$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om ´n punt*]

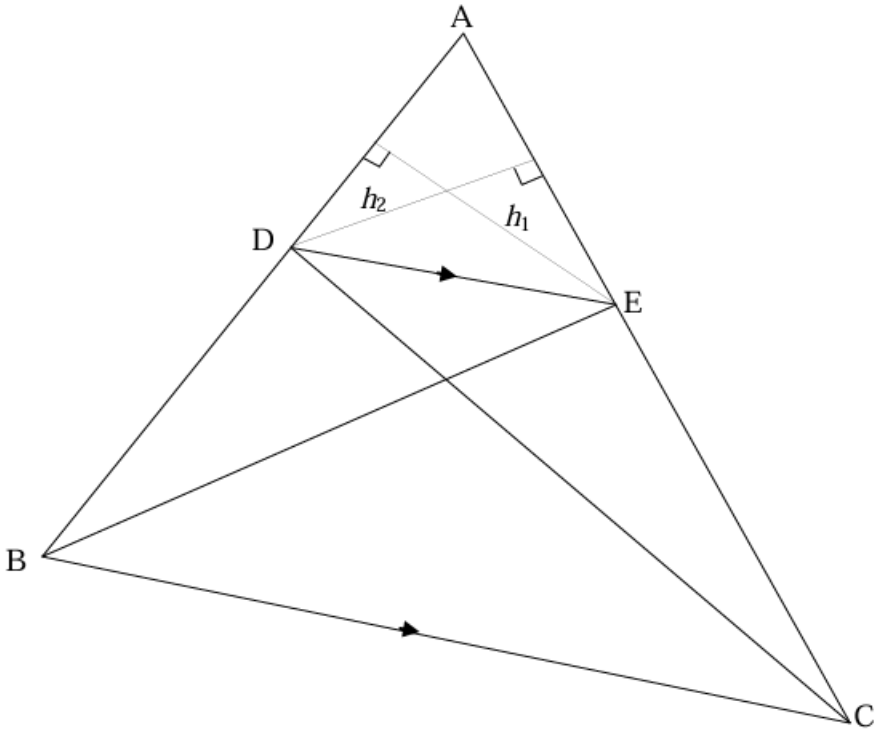
$\therefore \hat{M} = \frac{1}{2}\hat{O}_2$ [∠ at centre = 2 ×∠ at circumference]

$\therefore \hat{T}_2 + \hat{P}_1 = 180° - \hat{M}$ [sum of ∠s in Δ/*som ∠e van Δ*]

$\therefore \hat{T}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*]

$$= \frac{180° - \hat{M}}{2} = \frac{180° - \frac{1}{2}\hat{O}_2}{2} = \frac{180° - \frac{1}{2}(360° - x)}{2} = \frac{1}{4}x$$

$\therefore S\hat{T}M = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]



10.1

Constr: Draw h_1 from E $\perp$ AD and h_2 from D $\perp$ AE

Konstr: Trek h_1 *vanaf* E $\perp$ AD *en* h_2 *vanaf* D $\perp$ AE

Proof/*Bewys*:

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1} = \frac{AD}{DB}$$

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$$

But area ΔBDE = area ΔDEC [same base & height or DE || BC/ *dies basis & hoogte*; of DE || BC]

$$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

10.2.1

$\hat{V}_3 = x$ [Tans from same point/*raaklyne vanaf dieselfde pt*]

$\hat{R} = x$ [tan chord theorem/*raaklyn koordstelling*]

$\hat{W}_3 = x$ [corresp ∠s/*ooreenkomstige ∠e*; WT || RV]

10.2.2(a)

$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1]

W, S, T and V are concyclic/*is konsiklies*

WSTV is a cyclic quad [converse ∠s in the same segment/ *Omgekeerde ∠e in dieselfde segment*]

10.2.2(b)

$\hat{W}_2 = \hat{S}_2 = x$ [∠s in the same segment/∠e *in dies segment*]

$\hat{V}_1 = \hat{W}_2 = x$ [alt ∠s/*verwiss* ∠e ; WT || RV]

But $\hat{R} = x$ [proved in 10.2.1]

$\therefore \hat{R} = \hat{V}_1 = x$

$\therefore$ WR = WV [sides opp equal ∠s/*sye teenoor gelyke ∠e*]

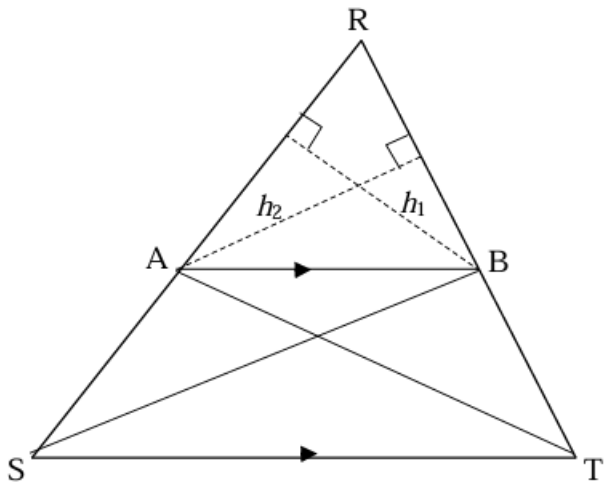
ΔWRV is isosceles/*is gelykbenig*

OR

	$\hat{S}_2 = \hat{W}_2 = x$ [∠s in the same segment] $\hat{W}_2 = \hat{W}_3 = x$ $\hat{W}_2 + \hat{W}_3 = \hat{R} + \hat{V}_1$ [ext ∠ of Δ] $\therefore\ \hat{V}_1 = x = \hat{R}$ $\therefore$ WR = WV [sides opp equal ∠s/*sye teenoor gelyke ∠e*] ΔWRV is isosceles/*is gelykbenig*
10.2.2(c)	In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\therefore$ ΔWRV ||| ΔTSV [∠, ∠, ∠] **OR/*OF*** In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\hat{W}_1 = S\hat{T}V = x$ [sum of ∠s in Δ/*∠e van* Δ] $\therefore$ ΔWRV ||| ΔTSV
10.2.2(d)	$\frac{RV}{SV} = \frac{WR}{TS}$ [ΔWRV ||| ΔTSV] $\therefore WR \times SV = RV \times TS$ $\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/*eweredighst*; WT ‖ RV] $\therefore WR \times SV = KV \times SR$ $\therefore RV \times TS = KV \times SR$ $\therefore \frac{RV}{SR} = \frac{KV}{TS}$ **OR/*OF*** In ΔRVS and/*en* ΔVKT $S\hat{V}R = \hat{K}_4$ [alt ∠s, WT ‖ RV] $S\hat{R}V = \hat{V}_3$ [proven] ΔRVS ||| ΔVKT [∠, ∠, ∠] $\therefore \frac{RV}{SR} = \frac{KV}{VT}$ but VT = ST [tans from same point] $\therefore \frac{RV}{SR} = \frac{KV}{TS}$

Paper H

9.1	$\hat{A}_1 = 40°$ [ext. ∠ of a cyclic quad / *buite ∠ van kvh*]
9.2	$\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $\hat{D} = 100°$ [opp ∠s of cyclic quad / *teenoorst. ∠e van kvh*] $\therefore \hat{C}_2 = 40°$ [sum of ∠s in Δ / *binne ∠e van* Δ] $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*] **OR/*OF*** $\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $A\hat{C}E = \hat{A}_2 + \hat{B}$ [ext ∠ of Δ / *buite ∠ van* Δ] $\therefore \hat{C}_2 = 40°$ $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*] **OR/*OF*** $\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $\therefore \hat{C}_3 = 180° - 46° - 80°$ [sum of ∠s in Δ / *binne ∠e van* Δ] $\therefore \hat{C}_3 = 54°$ $\therefore \hat{C}_2 = 180° - 86° - 54°$ [∠s on a str. line / *∠e op 'n reguitlyn*] $\therefore \hat{C}_2 = 40°$ $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*]



10.1 Construction: Join SB and TA and draw h_1 from B $\perp$ AR and h_2 from A $\perp$ RB

Konstruksie: Verbind SB en TA en trek h_1 vanaf B $\perp$ AR en h_2 vanaf A $\perp$ RB

Proof/*Bewys*:

$$\frac{\text{area } \Delta RAB}{\text{area } \Delta ASB} = \frac{\frac{1}{2}RA \times h_1}{\frac{1}{2}AS \times h_1} = \frac{RA}{AS}$$

$$\frac{\text{area } \Delta RAB}{\text{area } \Delta ABT} = \frac{\frac{1}{2}RB \times h_2}{\frac{1}{2}BT \times h_2} = \frac{RB}{BT}$$

area ΔRAB = area ΔRAB [common/*gemeenskaplik*]
But area ΔASB = area ΔABT [same base & height; AB || ST/ *dies. basis & hoogte; AB //ST*]

$$\therefore \frac{\text{area } \Delta RAB}{\text{area } \Delta ASB} = \frac{\text{area } \Delta RAB}{\text{area } \Delta ABT}$$

$$\therefore \frac{RA}{AS} = \frac{RB}{BT}$$

10.2.1

PR = RS [line from centre ⊥ to chord/ *lyn vanuit midpt. sirkel ⊥ op koord*]

PO = OW [radii / *radiusse*]

$\therefore \quad OR = \frac{1}{2}WS$ [midpt theorem/*midpt. stelling*]

$\therefore$ OR : WS = 1 : 2

OR/*OF*

$P\hat{S}W = 90°$ [∠ in semi circle/*∠ in halwe sirkel*]

$P\hat{R}O = 90°$ [given]

$\therefore P\hat{R}O = P\hat{S}W$

$\therefore$ RO || SW [corresp ∠s = / *ooreenk. ∠e =*]

OR/*OF*

[co-int. ∠s suppl / *ko-binne ∠e suppl*]

$\frac{PO}{OW} = \frac{PR}{RS}$ [prop theorem; RO || SW/ *lyn || een sy van* Δ]

PO = OW [radii / *radiusse*]

$\therefore$ PR = RS

$\therefore$ OR : WS = 1 : 2 [midpt theorem/ *midpt. stelling*]

OR/*OF*

ΔPRO and ΔPSW

$P\hat{S}W = 90°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*]

$P\hat{R}O = 90°$ [given]

$\therefore P\hat{R}O = P\hat{S}W$

$\hat{P}$ is common

$P\hat{O}R = P\hat{W}S$ [sum of $\angle$s in Δ/ *som van* $\angle$ *e in* Δ]

$\therefore \Delta PRO \,|||\, \Delta PSW$ [$\angle\angle\angle$]

$\therefore \frac{PO}{PW} = \frac{RO}{SW}$ [||| Δs / ||| Δe]

but $PW = 2\,PO$ [diameter = 2 radius/*middellyn = 2 radius*]

$\therefore \frac{RO}{SW} = \frac{PO}{2PO}$

$= \frac{1}{2}$

$\therefore$ OR : WS = 1 : 2

10.2.2

$\frac{OV}{VT} = \frac{RS}{ST} = \frac{1}{3}$ [prop theorem; RO || SW/ *lyn* || *een sy van* Δ]

$\frac{RS}{15} = \frac{1}{3}$

$RS = 5$ units

PR = RS = 5 units [line from centre $\perp$ to chord / *lyn vanuit midpt. sirkel* $\perp$ *op koord*]

$\therefore$ PT = 25 units

11.1

$\hat{D}_1 = E\hat{A}G = x$ [tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]

$\hat{C}_1 = \hat{D}_1 = x$ [tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]

$\hat{C}_4 = \hat{C}_1 = x$ [vert opp $\angle$s = / *regoorst. $\angle$e*]

$A\hat{F}G = E\hat{A}G = x$ [tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]

OR/*OF*

EA = EC [tans from common pt/ *raaklyne vanuit dies. punt*]

$\hat{C}_1 = E\hat{A}G = x$ [$\angle$s opp equal sides/ *$\angle$e teenoor gelyke sye*]

$\hat{C}_4 = \hat{C}_1 = x$ [vert opp $\angle$s = / *regoorst. $\angle$e*]

$\hat{D}_1 = E\hat{A}G = x$ [tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]

$A\hat{F}G = E\hat{A}G = x$ [tan-chord theorem $\angle$ *tussen raaklyn en koord*]

11.2	$\hat{D}_1 = A\hat{F}G = x$	
	$\therefore$ DC $\|\|$ FG	[corresp $\angle$s = / *ooreenk* $\angle e$ =]
	$\frac{AG}{AC} = \frac{AF}{AD}$	[prop theorem; DC $\|\|$ FG / *lyn* $\|\|$ *een sy van* Δ]
	$\therefore$ AG.AD = AC.AF	
	OR/OF	
	In ΔACD and ΔAGF	
	$\hat{A}_3$ is common	
	$A\hat{F}G = \hat{D}_1 = x$	[proved in 11.1 / *reeds bewys*]
	$\hat{C}_2 = A\hat{G}F = x$	[sum $\angle$ Δs/*binne* $\angle e$ Δ]
	ΔACD $\|\|\|$ ΔAGF	[$\angle\angle\angle$]
	$\frac{AC}{AG} = \frac{AD}{AF}$	[$\|\|\|$ Δs $\therefore$ sides in proportion / $\|\|\|$ Δe $\therefore$ *sye in dieselfde verhouding*]
	$\therefore$ AG.AD = AC.AF	
11.3	In Δ AGF and ΔABC	
	$\hat{G} = \hat{B}_2$	[$\angle$s in the same seg / $\angle e$ *in dies. segment*]
	$A\hat{F}G = \hat{C}_1 = x$	[proved in 11.1 / *reeds bewys*]
	$\hat{A}_3 = \hat{A}_2$	[sum of $\angle$s in Δ/*binne* $\angle e$ *van* Δ]
	Δ AGF $\|\|\|\Delta$ABC	[$\angle\angle\angle$]

11.4

$$\frac{GF}{BC}=\frac{AF}{AC} \qquad [\Delta AGF \,|||\, \Delta ABC]$$

$$\therefore GF=\frac{BC.AF}{AC}$$

$$\Delta ACD \,|||\, \Delta FGC \qquad [\angle\angle\angle]$$

$$\therefore \frac{AC}{GF}=\frac{AD}{FC}$$

$$\therefore AC=\frac{AD.FG}{FC}$$

$$\therefore GF=BC.AF\div\frac{AD.FG}{FC}$$

$$GF=BC.AF\times\frac{FC}{AD.FG}$$

$$\therefore GF^2=\frac{BC.FC.AF}{AD}$$

OR/*OF*

$$\Delta AGF \,|||\, \Delta ABC \qquad [\angle\angle\angle]$$

$$\frac{GF}{BC}=\frac{AF}{AC}$$

$$GF=\frac{AF.BC}{AC}$$

$$\Delta ACD \,|||\, \Delta AGF \qquad [\angle\angle\angle]$$

$$\frac{AD}{AF}=\frac{CD}{GF}$$

$$GF=\frac{AF.CD}{AD}$$

$$GF\times GF=\frac{AF.BC}{AC}.\frac{AF.CD}{AD}$$

$$\Delta FCD \,|||\, \Delta FAC \qquad [\angle\angle\angle]$$

$$\frac{FC}{FA}=\frac{CD}{AC} \qquad \text{from } |||\,\Delta\text{`s}$$

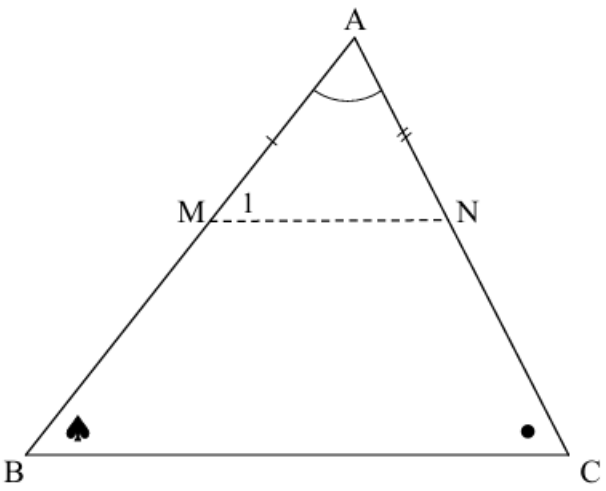
$$FC=\frac{CD.AF}{AC}$$

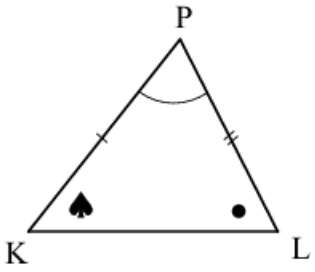
$$GF^2=\frac{AF.FC.BC}{AD}$$

Paper I

9.1.1	$\frac{HF}{EH}=\frac{GF}{GD}=\frac{2}{5}$	[line ‖ to one side of Δ/*lyn* ‖ *een sy van* Δ] **OR** [prop theorem; GH‖DE/*eweredigheidst.; GH ‖ DE*]
9.1.2	$\frac{EH}{EF}=\frac{DG}{DF}=\frac{5}{7}$ $\frac{EH}{21}=\frac{5}{7}$ $EH=15cm$ **OR/*OF*** $\frac{HF}{EF}=\frac{2}{7}$ $\frac{HF}{21}=\frac{2}{7}$ $HF=6cm$ $EH=21-6$ $EH=15cm$	[line ‖ to one side of Δ/*lyn* ‖ *een sy van* Δ] **OR** [prop theorem; GH‖DE/*eweredigheidst.; GH ‖ DE*] [line ‖ to one side of Δ/*lyn* ‖ *een sy van* Δ] **OR** [prop theorem; GH‖DE/*eweredigheidst.; GH ‖ DE*]
9.1.3	ΔFGH ||| **ΔFDE**	[∠∠∠]
9.1.4	$\frac{GH}{DE}=\frac{FH}{FE}$ [|||Δ's] $\frac{GH}{DE}=\frac{2}{7}$	**OR/*OF*** $\frac{GH}{DE}=\frac{FG}{FD}$ [|||Δ's]

9.2.1	$\hat{L}_2 = 90°$ [∠ in semi-circle/ *∠ in halwe sirkel*]	**OR** $\hat{L}_1 = \hat{R}_1$ [tan chord theorem/ *raaklyn-koordst.*]
	$\hat{R}_2 = 90°$ [∠ in semi-circle] *∠ in halwe sirkel*]	$\hat{R}_1 = \hat{P}$ [tan chord theorem] *raaklyn-koordst.*]
	$\therefore \hat{L}_2 = \hat{R}_2$	$\therefore \hat{L}_1 = \hat{P}$
	$\therefore$ LT ‖ PR [alt ∠s =/*verw. ∠e* =]	$\therefore$ LT ‖ PR [corresp. ∠s =/ *ooreenk. ∠e* =]
9.2.2	$\hat{L}_1 = \hat{R}_1$ [tan chord theorem/*raaklyn-koordst.*]	
	$\hat{L}_1 = \hat{O}_1$ [corresp. ∠s; LT ‖ OM/*ooreenk. ∠e; LT ‖OM*]	
	$\therefore \hat{R}_1 = \hat{O}_1$	
	$\therefore$ L, O, R and M are concyclic.	
	$\therefore$ LORM is a cyclic quadrilateral [converse ∠s in the same seg/ *omgekeerde ∠e in dies. sirkel segm*]	
9.2.3	$O\hat{L}R = \hat{M}_1$ [∠s in the same seg/*∠e in dieselfde segment*]	
	$2\hat{L}_3 = \hat{M}_1$ [∠ at centre = 2 ×∠ at circumference/ *midpt. ∠ = 2× omtreks ∠*]	
	$\therefore O\hat{L}R = 2\hat{L}_3$	
	$\therefore \hat{L}_4 = \hat{L}_3$	
	$\therefore$ LN bisects $O\hat{L}R$	





10.1 Construction: Draw line MN, where M and N are points on AB and AC respectively such that AM = PK and AN = PL.

In Δ AMN and ΔPKL

$\hat{A} = \hat{P}$ [given]
AM = PK [construction]
AN = PL [construction]
$\Delta AMN \equiv \Delta PKL$ [s$\angle$s]
$\therefore \hat{M}_1 = \hat{K}$
But $\hat{B} = \hat{K}$ [given]
$\therefore \hat{M}_1 = \hat{B}$
$\therefore MN \parallel BC$ [corresp $\angle$s =/*ooreenk. $\angle$e* =]
$\therefore \frac{AB}{AM} = \frac{AC}{AN}$ [line $\parallel$ one side of Δ/*lyn* $\parallel$ *een sy v* Δ]

But AM = PK and AN = PL

$\therefore \frac{AB}{PK} = \frac{AC}{PL}$

10.2.1 In ΔCFO and ΔCBD

$\hat{C} = \hat{C}$ [common]
$C\hat{F}O = C\hat{B}D$ [corresp $\angle$s; BD$\parallel$FO/*ooreenk. $\angle$e; BD$\parallel$FO*]
$C\hat{O}F = C\hat{D}B$ [sum of $\angle$s in Δ/*binne $\angle$e v* Δ]

OR

[corresp $\angle$s; BD$\parallel$FO/ *ooreenk. $\angle$e; BD$\parallel$FO*]
$\Delta CFO \parallel\parallel \Delta CBD$ [$\angle\angle\angle$]

10.2.2 $\frac{FO}{BD} = \frac{CO}{CD}$ [$\parallel\parallel$ Δs]
FO.CD = CO.BD
But $R\hat{D}O = F\hat{C}O$ [given]
$\therefore$ BD = BC [sides opp equal $\angle$s/*sye teenoor gelyke $\angle$e*]

$\therefore$ FO.CD = CO.BC

10.2.3

RD = 6 units [DR = RB]

$$\frac{RO}{RD} = \frac{3}{4}$$

$$\therefore RO = \frac{3}{4}(6)$$

RO = 4,5 units

OR ⊥ BD [line from centre to midpt of chord/ *midpt. sirkel; midpt koord*]

$$\therefore DO = \sqrt{6^2 + 4{,}5^2}$$ [Pythagoras]

DO = 7,5 units

$$\frac{BF}{BC} = \frac{DO}{DC}$$ [line || to one side of Δ/*lyn* || *een sy v* Δ]

OR/*OF*

[prop theorem; BD||FO/*eweredigheidst.;*BD||FO]

BC = BD = 12 units [sides opp equal ∠s/*sye teenoor gelyke ∠e*]

$$\therefore \frac{BF}{12} = \frac{7{,}5}{19{,}2}$$

$$BF = \frac{7{,}5 \times 12}{19{,}2}$$

$$BF = \frac{75}{16} \text{ units}$$

10.2.4

$$\tan R\hat{D}O = \frac{RO}{RD} = \frac{4{,}5}{6} = \frac{3}{4}$$

$$R\hat{D}O = 36{,}87°$$

$$F\hat{C}O = R\hat{D}O = 36{,}87°$$ [given]

$$\therefore A\hat{B}D = 73{,}74°$$ [ext ∠ of Δ/*buite ∠ v.* Δ]

OR/*OF*

$$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos D\hat{B}C$$

$$\cos D\hat{B}C = \frac{12^2 + 12^2 - 19{,}2^2}{2(12)(12)}$$

$$\cos D\hat{B}C = -\frac{7}{25}$$

$$D\hat{B}C = 106{,}26°$$

$$\therefore A\hat{B}D = 73{,}74°$$ [∠s on a straight line/*∠e op 'n reguitlyn*]

Paper J

8.1	$\hat{L} = 100°$	[ext ∠ cyclic quad = int opp ∠ / *buite ∠ kdvh = tos ∠*]
	OR/*OF*	
	$\hat{N}_1 = 80°$	[∠s on straight line]
	$\hat{L} = 100°$	[opp ∠s of cyclic quad]
8.2	$\hat{N}_1 = 80°$	[∠s on straight line/∠e *op reguitlyn*]
	$\therefore \hat{O}_1 = 160°$	[∠ at centre = 2 × ∠ at circumference/midpts ∠ = 2 omtreks ∠]
	OR/*OF*	
	reflex $K\hat{O}M = 200°$	[∠ at centre=2 × ∠ at circumference/*midpts∠ = 2×omtreks∠*]
	$\therefore \hat{O}_1 = 160°$	[∠s around a pt/ ∠e om 'n pt]
8.3	$\hat{M}_1 = 360° - (100° + 55° + 160°)$	[sum ∠*s* of quad/*som ∠e v vierhoek*]
	$\therefore \hat{M}_1 = 45°$	

9.1.1	∠ in semi-circle/∠ *in halfsirkel*
9.1.2	**Opp ∠s** of quad = **180°**/*Teenoorst ∠e v vierhoek=180°*
9.2.1	OF ⊥ AC [line from centre bisects chord/*lyn v midpt halv kd*] ∴ AC ∥ GO [co-interior/*ko-binne* ∠s = 180° **OR/*OF*** corresp/*ooreenkomstige* ∠s =]
9.2.2	$\hat{G}_1 = \hat{A}_2$ [corresp/*ooreenk* ∠s; AC ∥ GO] $\hat{A}_2 = \hat{B}_1$ [∠s in same segment/∠e *in dies segment*] $\therefore \hat{G}_1 = \hat{B}_1$ **OR/*OF*** $\hat{G}_1 = \hat{B}_2$ [ext ∠ cyclic quad/*buite ∠ koordevh*] but ΔABF ≡ ΔCBF [s, ∠, s] $\therefore \hat{B}_2 = \hat{B}_1$ $\therefore \hat{G}_1 = \hat{B}_1$
9.3	OF : FB = 3 : 2 DB = 2*r* ∴ DO = 5*k* and DF = 8*k* **OR/*OF*** $DF = 2r - \frac{2}{5}r = \frac{8}{5}r$ $\therefore \frac{DG}{DA} = \frac{DO}{DF} = \frac{r}{\frac{8}{5}r}$ [line ∥ side of Δ/*lyn ∥sy v* Δ] $\therefore \frac{DG}{DA} = \frac{5}{8}$

10.1	Tangent-chord theorem
10.2.1	$\hat{A}_2 + \hat{A}_3 = \hat{B}_1 + \hat{B}_2$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{S}_3 = \hat{B}_1 + \hat{B}_2$ [ext ∠ cyclic quad/*buite ∠ koordevh*] $\therefore \hat{S}_3 = \hat{A}_2 + \hat{A}_3$ ∴ AB ∥ ST [corresp/*ooreenk* ∠s =] **OR/*OF*** $R\hat{T}S = B\hat{A}S$ [ext ∠ cyclic quad/*buite ∠ koordevh*] $B\hat{A}S = A\hat{B}T$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore R\hat{T}S = A\hat{B}T$ ∴ AB ∥ ST [corresp/*ooreenk* ∠s =]

10.2.2

$\hat{B}_2 = x$ [tan chord theorem/*raakl – koordst*]

$x + \hat{T}_4 = \hat{B}_1 + \hat{B}_2$ [corresp/*ooreenk* $\angle^S$; AB // ST]

$\therefore \hat{T}_4 = \hat{B}_1$

$\hat{B}_1 = \hat{A}_1$ [tan chord theorem/*raakl – koordst*]

$\therefore \hat{T}_4 = \hat{A}_1$

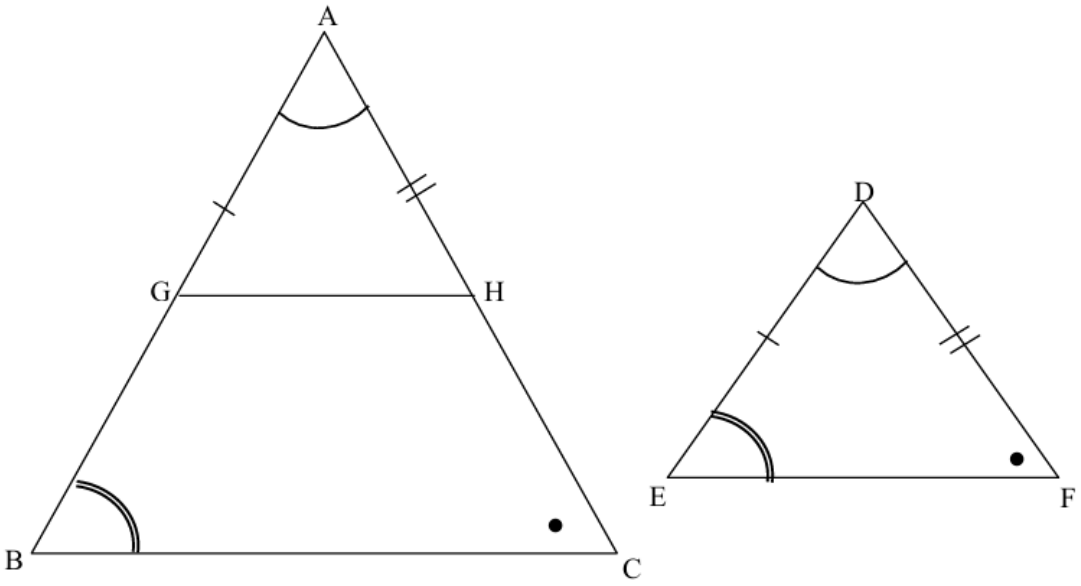
10.2.3

$\hat{T}_4 = \hat{A}_1$ [proven/*bewys in* 10.2.2]

$\therefore$ RTAP is a cyclic quadrilateral [line subtends = $\angle^s$]

Is 'n koordevierhoek [*lyn onderspan* = $\angle e$]

11.1



11.1

Constr: On sides AB and AC of ΔABC, mark points G and H respectively such that AG = DE and AH = DF. Draw GH/*Merk punt G en H op sy AB en AC van* ΔABC *onderskeidelik af sodanig dat AG = DE en AH = DF. Trek GH.*

Proof/*Bewys*:

$\Delta AGH \equiv \Delta DEF$ [s, $\angle$, s]

$\therefore A\hat{G}H = \hat{E}$

$= \hat{B}$ [$\hat{B} = \hat{E}$, given/*gegee*]

$\therefore GH \parallel BC$ [corresp/*ooreenk* $\angle^s$ =]

$\therefore \frac{AG}{AB} = \frac{AH}{AC}$ [line $\parallel$ side of Δ / *lyn* $\parallel$ *sye v* Δ]

$\therefore \frac{DE}{AB} = \frac{DF}{AC}$ [constr/*konstruksie*]

11.2.1(a)

$AP = PC$ [diag $\|^m$ bisect each other/*hoekl* $\|^m$ *halveer mekaar*]

But $TP = PS$ [given/*gegee*]

$AP - TP = PC - PS$

$\therefore AT = SC$

11.2.1(b)

In ΔPSR and ΔPBA:

$\hat{P}_1 = \hat{P}_3$ [vertically opp $\angle^s$ / *regoorst* $\angle e$]

$\hat{B}_1 = \hat{S}_1$ [$\angle^s$ in same segment/*$\angle e$ in dies segment*]

$\therefore \Delta PSR \,|||\, \Delta PBA$ [$\angle, \angle, \angle$]

OR/*OF*

In ΔPSR and ΔPBA:

$\hat{P}_1 = \hat{P}_3$ [vertically opp $\angle^s$ / *regoorst* $\angle e$]

$\hat{B}_1 = \hat{S}_1$ [$\angle^s$ in same segment/*$\angle e$ in dies segment*]

$\hat{A}_2 = \hat{R}_1$ [sum $\angle^s \Delta$ / *som* $\angle e \Delta$]

$\therefore \Delta PSR \,|||\, \Delta PBA$ [$\angle, \angle, \angle$]

11.2.2(a)

$\frac{PR}{PA} = \frac{PS}{PB}$ [$|||$ s]

$\therefore \frac{PR}{PA} = \frac{TR}{AD} = \frac{PS}{PB}$ [given $\frac{PR}{PA} = \frac{TR}{AD}$]

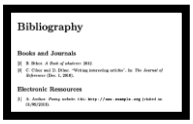
$\therefore \frac{PR}{PA} = \frac{TR}{AD} = \frac{TP}{PD}$ [$PS = TP; PB = PD$]

$\therefore \Delta RPT \,|||\, \Delta APD$ [sides of Δ in prop/*sye v* Δ *in dies verhouding*]

11.2.2(b)

$\hat{T}_1 = \hat{D}_2$ [$|||$ s]

$\therefore$ ATRD is a cyclic quad [converse: ext $\angle$ of cyclic quad/ *Omgekeerde buite* $\angle$ v koordevh]



BIBLIOGRAPHY

1	Past Exam Memos (2013 – 2026)